Statistical investigations of flow structures in different regimes of the stable boundary layer

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Abstract

A combination of methods originating from non-stationary timeseries analysis is applied to two datasets of near surface turbulence in order to gain insights on the non-stationary enhancement mechanism of intermittent turbulence in the stable atmospheric boundary layer (SBL). We identify regimes of SBL turbulence for which the range of timescales of turbulence and submeso motions, and hence their scale separation (or lack of separation) differs. Ubiquitous flow structures, or events, are extracted from the turbulence data in each flow regime. We relate flow regimes characterised by very stable stratification but different scales activity to a signature of flow structures thought to be submeso motions.

1 Introduction

The representation of the stable boundary layer (SBL) presents ongoing challenges, and modelling challenges increase with increasing stability (Sandu et al., 2013). Among the more unknown situations are small wind speed scenarios in which the turbulence is weak and does not show significant dependence on the stratification. In such weak wind situations, turbulence typically becomes non-stationary and a spectrum of motions on the so-called sub-mesoscales is found to bridge the scale gap between the largest turbulence is found to be enhanced by these submeso motions (Mahrt and Thomas, 2016; Sun et al., 2015; Cava et al., 2016). Better understanding of the non-stationary enhancement mechanism is a necessary step towards improved SBL turbulence parameterisation.

Recent approaches focus on distinguishing regimes in which turbulence behaves differently. Based on observations, Sun et al. (2012) identify a height dependent wind speed threshold that separates a regime in which turbulence increases slowly with increasing wind speed from a regime where turbulence increases rapidly with the wind speed. The weak turbulence, strongly stable regime is found to include cases where local shear-generated eddies are too small to interact with the ground and turbulence is not related to the bulk shear anymore.

Theoretical findings also predict the appearance of two regimes based on the hypothesis that continuous turbulence requires the turbulence heat flux to balance the surface energy demand resulting from radiative cooling (van de Wiel et al., 2012a,b, 2017). A radiative heat loss that is stronger than the maximum turbulent heat flux that can be supported by the flow with a given wind profile will lead to the cessation of turbulence (van de Wiel et al., 2012a). This concept is used by van Hooijdonk et al. (2015) to show that the shear over a layer of certain thickness can predict SBL regimes when sufficient averaging of data is considered.

The very stable regime is however more prone to be dominated by apparently random, sub-mesoscale wind accelerations that can generate local turbulence and lead to highly non-stationary flows (Acevedo et al., 2015). Such local accelerations have been revealed by released fog elements and fine scale temperature measurements from fibre optic distributed temperature sensing (Zeeman et al., 2014). Numerical studies have shown that finite perturbations imposed on the flow after cessation of turbulence can suffice to act as a regenerating mechanism for turbulence (Donda et al., 2015). Donda et al. (2015) further found a strong sensitivity of the turbulence recovery to the timing and amplitude of added perturbations, thereby motivating the need for better characterisation of sub-mesoscale motions and their effect on turbulence. Statistical analyses of the hydrodynamical equilibrium properties of the SBL flow revealed that the very stable regime is prone to larger memory effects and needs to be represented by high order closure models or stochastic processes (Nevo et al., 2017).

Despite numerous case studies highlighting the local shear generation of turbulence due to wind speed accelerations connected to submeso motions (Sun et al., 2004; Román Cascón et al., 2015; Mortarini et al., 2017), the general understanding of non-turbulent motions on sub-mesoscales remains very limited. Analyses of the propagation direction of submeso motions revealed no tendency to follow the mean wind direction (Lang et al., 2017) and highlighted the difficulty to understand the origin of such features of the flow. To extend case studies to more general observations, Kang et al. (2014) developed a method to extract non-stationary motions from turbulent timeseries, regardless of the physical origin of the flow motions. Non-stationary flow structures from SBL data were subsequently categorised into three classes with similar characteristics (Kang et al., 2015). The smoothest, wave-like structures were typically associated with stronger wind, active turbulence and weak stability. The two other classes associated with higher stratification were found to have predominantly sharp structures and one of them included step-like structures that were attributed to microfronts.

To further investigate the local shear generation of eddies, Vercauteren and Klein (2015) followed a data driven approach to identify regimes based on the relationship between turbulence and local wind variations on the sub-mesoscales. Regime identification was based on the FEM-BV-VARX clustering procedure (Horenko, 2010; O'Kane et al., 2016), which allows identification of regime modulation by external factors. The automatic procedure was developed to isolate periods in which turbulence is related to local acceleration of the flow due to propagating non-turbulent motions on the sub-mesoscales. Further analysis revealed that one of two identified types of submeso-influenced regimes gathered cases in which a scale gap separated the smallest sub-mesoscales from the largest turbulence scale. In the second such regime, sub-mesoscales

and turbulent scales seemed to overlap (Vercauteren et al., 2016). Based on this classification of submesoinfluenced flow regimes, the present study will characterise the statistics of submeso motions that occur in flow regimes characterised by different scale activity. The extraction of submeso motions will be based on the Turbulent Event Detection (TED) method proposed by (Kang et al., 2015). The questions that will be addressed are the following: Is there a preferred type of submeso motion that interacts with turbulence? And does the frequency and type of submeso-motions change depending on the regimes of SBL turbulence?

2 Data

Our study is based on sonic anemometers measurements from the Snow Horizontal Array Turbulence Study (SnoHATS, Bou-Zeid et al. (2010)) and from the Fluxes over Snow Surfaces II (FLOSSII, Mahrt (2010)) datasets. The SnoHATS dataset was collected over a large flat glacier on top of a mountain range. The FLOSSII dataset was collected over a locally flat basin between two mountain ranges and includes several snow covered periods. Some measures of turbulence and submeso activity are given for both sites in Table 1.

2.1 SnoHATS

The data was collected over the Plaine Morte Glacier in the Swiss Alps from February to April 2006, at 2750m elevation ((Bou-Zeid et al., 2010), data collected by the EFLUM laboratory at EPFL). The large flat glacier ensures long periods of stable stratification, and measurements were taken at a height varying between 2.82 m and 0.62 m, depending on snow accumulation. The setup, shown in Fig. 1, consists of two vertically separated horizontal arrays of sonic anemometers, with a total of 12 sonic anemometers (Campbell Scientific, model CSAT3). The vertical separation between the upper and lower array is 77 cm (82 cm after March 17), while the horizontal separation between the instruments is 80 cm. The data analysis was restricted to wind directions within a $\pm 60^{\circ}$ angle relative to the streamwise sonic axis (corresponding to easterly winds), ensuring that data are not affected by the structure supporting the instruments. The resulting fetch consists of 1500m of flat snow. After removing data with unfavourable wind angles (outside the selected $\pm 60^{\circ}$ range) or low quality (snow-covered sonics, power outages, etc), about 15 non-continuous days of data remained available for the analysis. The 20Hz raw data were preprocessed and conditioned using axis rotations to correct for the yaw and pitch misalignments of the sonics, linear detrending and density correction.



Figure 1: Setup of the SnoHATS field campaign. Left: Side view with the 12 instruments. Right: View in the direction of measurements showing the 1.5 km fetch.

2.2 FLOSSII

The data was collected from 20 November 2002 to 4 April 2003 over a locally flat surface south of Walden, Colorado, USA, in the Arapaho National Wildlife Refuge. The surface consists of matted grass with brush

Table 1: Site characteristics. Averaged values of the 30-min records for: the standard deviation of the vertical velocity fluctuations σ_w , the wind speed V, the sub-mesoscale wind velocity \hat{V} (defined formally in Section 3.1), the percentage of the time where the submeso-velocity scale is greater than the speed of the 30-min averaged wind vector, and the submeso cross-stream velocity variance σ_{vM}^2 (defined formally in Vickers and Mahrt (2007), equation (1)). The average include all instruments at the site.

Site	σ_w [m/s]	σ_w/V [-]	$V\\[m/s]$	\hat{V} [m/s]	$ \hat{V}/V > 1 \\ [\%] $	$\begin{array}{c} \sigma_{vM}^2 \\ [-] \end{array}$
SnoHATS FLOSSII	$\begin{array}{c} 0.18\\ 0.31 \end{array}$	$\begin{array}{c} 0.09 \\ 0.06 \end{array}$	$2.68 \\ 5.26$	$0.70 \\ 0.74$	$\begin{array}{c} 4.6\\ 10.18\end{array}$	$\begin{array}{c} 0.72 \\ 0.35 \end{array}$

upwind about 100 m. The grass was often covered by a thin snow layer during the field program. A tower collected measurements at 1, 2, 5, 10, 15, 20 and 30 m with Campbell CSAT3 sonic anemometers and the data from the second level (2 m) are used to identify flow regimes, extract and characterise events. The choice of the 2 m level is made to be similar to the SnoHATS data as well as to be enough above the ground to avoid dissipation of structures by small-scale turbulence near the surface. The data set was quality controlled and segments of instrument problems and meteorologically impossible values were eliminated (Larry Mahrt, personal communication). We restrict the analysis to night time data, taken between 18:00 and 7:00 (Local time). Flow regime identification based on the FEM-BV-VARX clustering methodology (see Section 3.1) ideally requires continuous data, however the dataset will consist of continuous night time data separated by gaps during the day. In order to maximise continuity of the dataset, nights with data gaps corresponding to more than 80 minutes (12 nights) as well as nights with wind flowing regularly through the measurement tower for periods longer than 5 minutes (51 nights) were removed from the analysis. The resulting 68 nights left for analysis have data gaps shorter than 1 minute and are deemed mostly uncontaminated. The short gaps are linearly interpolated. The 60Hz raw data is double rotated in to the mean wind direction based on 30 minutes average.

3 Methods

Our analyses of flow structures in the SBL are based on two complementary methods. In a first step, flow regimes are identified based on dynamical interactions between different scales of motion, using a dataclustering methodology based on a finite element, bounded variation, vector autoregressive factor method (FEM- BV-VARX) introduced by Horenko (Horenko, 2010; O'Kane et al., 2013). We hypothesise that the turbulence will sometimes be mainly controlled by the wind variability on sub-mesoscales (typically in weak wind, strongly stable situations) and our goal is to automatically detect periods in which the sub-mesoscale wind velocity influences the turbulence (Vercauteren and Klein, 2015). In the second step, we apply the Turbulent Event Detection method introduced by Kang et al. (2014, 2015) to detect events in noisy timeseries. The type of turbulent event occurring will be analysed in each of the FEM-BV-VARX identified flow regime separately, thus giving indication on the type of submeso motions occurring in each of the flow regimes detected based on scale interaction properties.

3.1 Classification of flow regimes

In this section we briefly review the mathematical framework used to classify the flow regimes in terms of their scale interactions properties. For the full details of the mathematical framework, we refer to Horenko (2010), while further details on its application to SBL flow regime classification can be found in Vercauteren and Klein (2015).

The FEM-BV-VARX method allows the user to detect regime modulation by external variables. When using the FEM-BV-VARX method to identify the regimes, we thus assume that the evolution in time of the vertical velocity fluctuations $\sigma_w = \sqrt{w'w'}$ (where the overbar denotes an averaging period of 1 minute and the prime denotes deviations from the average) can be approximated by several locally stationary statistical processes that are influenced by the sub-mesoscale horizontal wind velocity \hat{V} , defined on scales between 1 and 30 min. The sub-mesoscale mean wind speed is defined formally as

$$\hat{V} = \sqrt{\hat{u}^2 + \hat{v}^2},\tag{1}$$

where $\hat{\phi} = \overline{\phi} - [\phi]$, the overbar denotes a 1-min averaging time and the square brackets denote a 30-min averaging time, such that these fluctuations represent the deviations of the 1-min sub- record averages from the 30-min average. The definition of sub-mesoscales is made because those are scales that typically correspond to non-turbulent motions in weak-wind SBL flows (Mahrt et al., 2012a). Furthermore, the choice of the 1 min averaging time for the vertical velocity variance is a compromise between minimising loss of flux by larger-scale turbulent motions with windy conditions and minimising the contamination of the computed fluctuations by non-turbulent motions for weak- wind more-stable conditions. The statistical processes representing the time evolution of σ_w in the FEM-BV-VARX framework are vector autoregressive models with exogenous factors (VARX) and the sub-mesoscale wind velocity \hat{V} is considered as the exogenous factor. Our analyses showed that models which included an autoregressive part did not give any reproducible solutions to the clustering problem. Hence we restrict our search to models including only the exogenous part:

$$\sigma_{w:t} = \mu(t) + B_0(t)\dot{V}_t + \dots + B_p(t)\dot{V}_{t-p\tau} + C(t)\epsilon_t,$$
(2)

where the process $\sigma_{w;t}$ is the time evolution of the 1-min vertical velocity variance measured at one location; the external factors \hat{V}_t are the time evolution of the streamwise velocity on scales between 1 and 30min. $\epsilon_t \colon [0,T] \to \mathbb{R}^h (h \ll n)$ is a noise process with zero expectation, the parameters μ , B and C are timedependent matrix coefficients for the process and p is the memory depth of the external factor which needs to be estimated. The number of statistical processes corresponds to the number of clusters; the assumption of local stationarity of the statistical process is enforced by setting a persistency parameter C_p , which defines the maximum allowed number of transitions between K different statistical processes (corresponding to different values of the matrix coefficients). The cluster states are indicated by a cluster affiliation function, which is calculated by the procedure. The reader is referred to Horenko (2010) for information regarding the minimisation procedure used to solve the clustering problem. User defined parameters and their choice is discussed in Section 4.1.

3.2 Turbulent events detection

The time series analysis methodology for turbulent event detection (TED) derived by Kang et al. (2014) aims at identifying non-stationary events or flow patterns in noisy time series. Instead of detecting signatures of known flow patterns in time series, the TED method detects flow structures as events that are significantly different from noise. In the context of time series resulting from turbulent quantities, the noise is taken as white and red noise. Indeed, statistical descriptions of turbulence as first suggested by Kolmogorov (1941) and Obukhov (1941) lead to the formulation of stochastic models for the turbulent observables such that in the inertial subrange, Lagrangian velocities can be modelled by a Langevin equation (or Ornstein-Uhlenbeck process) with suitable drift and noise terms (Thomson, 1987):

$$\mathrm{d}u = -\frac{u}{T}\mathrm{d}t + \sqrt{C_0\varepsilon}\,\mathrm{d}W\,,\tag{3}$$

where u is the velocity (or a turbulent observable), T is the Lagrangian decorrelation time scale, C_0 is a universal constant and ε is the the mean dissipation; dW are increments of a Wiener process. As shown in Faranda et al. (2014), this model is in fact equivalent to an autoregressive process of order one (AR(1)) process (also known as red noise):

$$u_t = \phi \, u_{t-1} + \psi_t \,, \tag{4}$$

where t is a discrete time label, $\phi = 1 - \Delta t/T$, and ψ_t represents independent variables, normally distributed.

In the SBL, gravity waves, transient drainage flows and other flow structures on sub-mesoscale will typically superimpose on the turbulence or affect its intensity, thereby inducing non-stationarity and hence departures from the idealised inertial subrange Langevin model (3) or AR(1) model (4) (Nevo et al., 2017). This is the core idea of the TED method: in a first step, sequential subsequences of the time series x(t) of

turbulent observables are analysed using a sliding window of predefined length-scale l. The q-th subsequence is thus:

$$x_q(t) = \{x(t_q), \dots x(t_{q+l-1})\},$$
(5)

where $1 \le q \le (m - l + 1)$ and m is the length of the time series x(t). Events are defined as subsequences that are significantly different from white noise or from an AR(1) process. In practice, an AR(1) model is fitted to each detrended subsequence $x_q(t)$ and a test is performed on the model residuals to see whether they are uncorrelated. If this is not the case (i.e. if the residuals are not white noise), then $x_q(t)$ is defined as a potential event. Additionally, non-stationary subsequences that exhibit a structural break are considered as potential events. Note that the noise process is not removed from the subsequence, meaning that the potential event consists of the raw subsequence.

The TED approach assumes that the typical duration of an event is known, but its form is unknown. In the context of detecting submeso motions, this is appropriate since submeso motions can take many different forms that are poorly known, but the typical duration of events is on the scale of minutes to an hour. A complementary approach to detect events in noisy timeseries is to assume that the form of events is known, while the duration is unknown. This approach is proposed in (Lilly, 2017) where wavelet elements embedded in noise can detect isolated events of a known form.

In the second step of the TED method, statistical feature parameters are calculated for the detected events and are used to cluster the events into several groups sharing similar features. The features that are considered for each subsequence are the standard deviation σ of the subsequence, its kurtosis, skewness, the absolute difference between averages of the first and second half (which will be denoted as HD), nonsmoothness (or sharpness), two test statistics (of the Phillippe-Peron (PP) test and the Zivot and Andrews (ZA) test described in Kang et al. (2014) and maximum, minimum, and kurtosis of the first-order difference of the events. The reader is referred to Kang et al. (2014); Kang et al. (2015) for full details on the TED method. Time scales considerations are discussed next.

3.3 Averaging time

In the TED method, the length of the time window l has to be predefined. The choice of scale for the block averages of the turbulent observables will define the time increments of the AR(1) model in Eq. 4. Hence the averaging scale should be chosen such that the increments fall within the range of scales of inertial turbulence. As shown by the extended multiresolution decomposition (MRD) analyses in Vercauteren et al. (2016), scales faster than approximately 5-10 seconds exhibit fluctuations characteristic of isotropic turbulence and block averaging within this time range represents an appropriate choice.

In order to have results comparable to the analyses of Kang et al. (2015), we thus consider their choice of block averages of 6 seconds and a window length of 120 points (12 minutes). As discussed in Kang et al. (2014), events can be detected on multiple overlapping windows, such that the maximal event length is not limited to one window length. The extended MRD results in Vercauteren et al. (2016) highlight non-turbulent fluctuations in the range of 50 seconds to 20-30 minutes, depending on the flow regime. The window length of 12 minutes, with possibilities of longer events through overlapping windows, is hence deemed appropriate.

For the analysis of multiple scales, according to Kang et al. (2014), keeping l constant and block averaging the time series to a desired scale leads to better results. This is due to the fact that the test statistic applied for the white noise test depends on l and keeping l constant returns consistent results for all scales. Tests varying the size of block averages between 1 and 15 seconds (while keeping l = 120 points for consistency) showed a large sensitivity of the event detection to the choice of scales, highlighting the difficulty of using automatic methods for analyses of submeso motions (Kaiser, 2016). Nevertheless, we present our results using the aforementioned time scales - deemed physically appropriate by the MRD analyses - in the following section.

4 Results

4.1 Parameters selection for flow regime classification

The FEM-BV-VARX framework is used to classify flow regimes in the SnoHATS and in the FLOSSII datasets. The turbulence data under consideration in Eq. 2 is the time evolution of the 1-min vertical velocity variance measured at one location, $\sigma_w(t)$; the external factor is the time evolution of the streamwise velocity on scales between 1 and 30min at that location, $\hat{V}(t)$. For the SnoHATS dataset, this location corresponds to one of the 12 sonic anemometers (results showed little sensitivity to the choice of instrument). For the FLOSSII dataset, the location is chosen as the second level (2 m), corresponding to a height similar to the SnoHATS data.

User defined parameters include the maximum memory depth for the forcing variable p, the number of clusters K and the persistency parameter C_p , which limits the number of transitions between the clusters. The maximum memory lag that we use in this model is determined by a priori calculation of the partial autocorrelation function (pacf) for the variables σ_w and \hat{V} (Brockwell and Davis, 2002). The correlation between the time series drops on average after a few minutes, and was set to p = 3 for the SnoHATS dataset and p = 6 in the FLOSSII dataset (based on the average pacf over 68 nights). To determine the optimum number of K and C_p , multiple models are fitted for varied values of the parameters K and C_p . The Akaike Information Criteria (AIC, see Brockwell and Davis (2002)) was used in Vercauteren and Klein (2015) to select the optimal number of cluster states K = 4 and the persistency parameter $C_p = 20$ for the SnoHATS data.

For the FLOSSII dataset however, the AIC exhibits asymptotic behaviour towards zero for all models in the investigated parameter space (K = 2, 3, 4, 5, 7 and $C_p = [2, 302]$) and cannot be used as a selection criteria. This can be explained by the fact that FLOSSII data have comparably few occurrences of strongly stable regimes, where the dynamics of \hat{V} is strongly related to that of σ_w . Since the influence of \hat{V} is the only factor considered in the model (Eq. 2), when the influence of the dynamics of \hat{V} on that of σ_w is small, the algorithm will tend to cluster the time series based only on the mean of σ_w ($\mu(t)$ in Eq. 2). This is due to the fact that the method is based on minimising the Euclidean distance between the signal σ_w and a series of piecewise stationary deterministic statistical models (Horenko, 2010). If the exogenous part of the model is small, the mean becomes the most important contribution. The SnoHATS data includes many more strongly stable periods where the dynamics of \hat{V} is strongly related to that of σ_w , and the AIC could be used for model selection.

To select the model we therefore apply the following strategy: The quality of the model can be described in terms of how much correlation between \hat{V} and the model residual is left after the modelling, and on what amount of variance the model is explaining relative to the measured signal. By observing the change of these two quantities over the parameter space, we find that increasing the parameters beyond K = 3 and $C_p = 150$ does not reduce the correlation in the residuals, neither increase the modelled variance. Thus the choice of K = 3 and $C_p = 150$ is considered as an optimal model. The model fitting procedure was repeated five times for each model within the parameter space to evaluate the consistency of the solutions (resulting in total amount of identified models equal to 7500). The statistical models obtained with K = 3 and $C_p = 150$ have a high degree of reproducibility. Over five repeated minimisation procedures for the FLOSSII data, the cluster affiliation function is consistent (or equal) to a degree of 90%. Further increasing the parameters is leading to drop in the consistency of the cluster affiliation function below 50%. In other words, the results are getting unreproducible.

The amount of variance of $\sigma_w(t)$ explained by the VARX model in the three clusters is 0.8%, 3% and 9.5%, pointing to only one cluster in which the influence of the sub-mesoscale wind velocity (the external factor influencing the dynamics of σ_w) is significant. Analysis of the model residuals in the three clusters however showed that the error distribution in the cluster corresponding to the largest explained variance of 9.5% was not Gaussian. This cluster gathers the smallest values of σ_w and has the most interaction between sub-mesoscales and vertical velocity fluctuations and we want to classify the dynamical interactions more accurately. Therefore, we select the time series in this specific cluster and classify it with the FEM-BV-VARX methodology further into two distinct clusters. This strategy leads to error distributions that are closer to normally distributed in the two subsequent clusters. Selecting only those periods of larger dynamical interactions between σ_w and \hat{V} enables a second level clustering which differentiates the dynamical interactions interactions and not just the mean turbulent state. As a result, the data are clustered in four distinct regimes in both datasets.

Flow characteristics are given for each cluster in Table 2. The gradient Richardson number

$$Rib = (g/\Theta_0) \frac{\partial \theta/\partial z}{\left(\partial \bar{\mathbf{V}}/\partial z\right)^2},\tag{6}$$

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Site	Flow regime	Ri	$V(m.s^{-1})$	$\hat{V}(m.s^{-1})$	$\% \frac{\hat{V}}{V} > 1$
SnoHATS	S1	$0.02 \ (0.01, 0.05)$	5.6(3.9, 7.1)	1.6(1.0, 2.4)	7.8
SnoHATS	S2	0.12(0.06, 0.23)	4.2(3.2, 5.2)	0.8(0.5, 1.2)	4.8
SnoHATS	S3	0.29(0.10, 0.80)	2.1(1.3, 3.0)	$0.8 \ (0.5, 1.3)$	12.8
SnoHATS	$\mathbf{S4}$	$0.67 \ (0.28, 1.76)$	1.9(1.1, 2.7)	0.4(0.3, 0.7)	6.5
FLOSSII	F1	$0.03 \ (0.02, 0.04)$	9.0(7.8, 10.3)	$0.59 \ (0.27, 1.07)$	0.19
FLOSSII	F2	$0.07 \ (0.05, 0.10)$	5.7(4.8, 6.7)	$0.43 \ (0.20, 0.79)$	0.31
FLOSSII	F3	0.14(0.10, 0.24)	3.6(2.7, 4.4)	$0.46\ (0.20, 0.90)$	5.45
FLOSSII	F4	$0.58\ (0.26, 1.55)$	$1.3\ (0.7, 2.1)$	0.40(0.18, 0.76)	16.63

Table 2: Flow characteristics in each flow regime. Richardson number, wind speed, sub-mesoscale wind speed (Median (first and third quartiles) given for each value), and the percentage of times where the submeso wind velocity exceeds the wind velocity. All values are based on one-minute averaged data.

is used for indicative assessment of the stability properties in each regime. In (6), g is the gravity acceleration, θ is the potential temperature (Θ_0 being the averaged one over the record), \mathbf{V} is the wind vector, and the overline denotes a time average of one minute. The vertical gradients are calculated using the averages of the upper and lower sensors for the SnoHATS data, and 1 and 10m levels for the FLOSSII data. The median and quartiles of Ri in each cluster (Table 2) shows that weakly stable periods are separated from strongly stable periods by the FEM-BV-VARX procedure, with however large overlaps in the distributions of Riin the different clusters. Since the classification is based on the modulation of the turbulence by submeso motions, this separation strengthened the hypothesis that modulation of the turbulence by submeso motions differs between the weakly and very stable regimes.

4.2 Scale interaction properties

In each identified regime of near-surface SBL turbulence, the transport properties of different scales of motion are assessed using a multiresolution flux decomposition (MRD) (Vickers and Mahrt, 2003). The scales activity is shown in Fig. 2, based on MRD analyses. MRD heat flux cospectra are calculated on periods of 30 minutes, and results are shown for the average and standard deviations over all periods within a given flow regime. In all flow regimes, the averaged MRD show an increased negative contribution with increasing scales until a maximum, followed by a decrease and finally crossing the zero flux line. This is the signature expected from the turbulent contribution and the scale at which the MRD heat flux first reaches zero is typically used to estimate the scale of the spectral gap between turbulence and larger scale motions, or the averaging scale required to sample the turbulent heat flux.

The MRD of the heat fluxes show that while the averaged impact of a multitude of submeso contributions to the heat fluxes are very small, individual submeso contributions can be more important than turbulent contributions. This is apparent from the standard deviations of the largest scales of the MRD, which are in some cases larger than the amplitude of the maximum turbulent contribution. As the stability is increasing going from Regime F1 to F4 (Fig. 2, right panels) or S1 to S4 (Fig. 2, left panels), the activity of the turbulent scales in the overall heat transport is reducing. The submeso activity highlighted by the standard deviation lines however tends to increase. As such, the relative contribution of the sub-mesoscales increases with increasing stability. As the turbulence is collapsing, a state is reached where the magnitude of the heat flux due to sustained turbulent scales is overpassed by the local activity of the submeso motions (Fig. 2, Regime F4 and Regime S4). In the most stable regimes the local activity of submeso motion can be greater than that of the turbulent scales, thereby having the dominant contribution to the heat transport.

The conditional averages of the MRD heat fluxes based on flow regimes in the SnoHATS dataset (Fig. 2, left panels) highlight a strongly stable regime in which the standard deviation of the contributions from submesoscales is large, while the contributions from turbulent scales are small (Regime S4). This denotes a scale separation between the turbulent and sub-mesoscales. In another strongly stable flow regime (Regime S3), the turbulent and submeso contributions appear to overlap in scales, as highlighted by the continuously increasing standard deviations of the contributions of increasing scales. In a context of a scale separation between

Cluster	1	2	3
Non-Smoothness	1.290	1.388	1.558
Sigma (Kelvin)	0.391	0.469	0.268
HD (Kelvin)	0.100	0.270	0.094
Kurtosis	3.00	2.464	6.742
Skewness	0.496	0.481	1.608
Diff max (Kelvin)	0.715	0.524	0.660
Diff min (Kelvin)	-0.703	-0.589	-0.754
Diff Kurtosis	3.696	5.609	9.758
PPstat	-4.976	-3.156	-4.016
ZAstat	-5.468	-4.151	-4.649

Table 3: Centroids of the events of the SnoHATS temperature time series in each cluster based on k-means clustering. The parameters are defined in Section (3.2).

turbulence and submeso motions, the turbulence may have time to equilibrate to local shear accelerations due to submeso motions, but when the scales overlap, the turbulence cannot equilibrate to the continuously evolving forcing. Extended MRD results (Nilsson et al., 2014) were analysed in Vercauteren et al. (2016) and highlighted differences in the scales of generation of turbulence by submeso motions in the four flow regimes. Specifically, in the regimes of strong influence of submeso forcing S3 and S4, the results suggested a likely direct transfer of energy from the sub-mesoscale horizontal velocity fluctuations to turbulent vertical velocity fluctuations. Similar to the results of the MRD heat flux, the strongly stable regime S4 was characterised by a scale gap between submeso horizontal fluctuations and turbulence, whereas flux variability is more continuous in scale in the other strongly stable regime S3.

Having separated regimes where submeso activity is found to relate to turbulent vertical velocity fluctuations in both datasets, the next section presents the submeso motions identified by the TED method in each of the classified flow regimes.

4.3 Event extraction and clustering

Using the window size l = 120 and 6s averaged data, the first step of the TED method yields 1793 events in the SnoHATS temperature time series obtained by all 12 sonic anemometers, and 702 events in the FLOSSII 2-m temperature time series. A detailed analysis of the events extracted and clustered from the 2-m temperature time series of the FLOSSII data is given in Kang et al. (2015) and is not repeated here.

In the event clustering step of the TED method, each event is first represented using the feature vector with the features listed in Section (3.2) and Table (3). Principal component analysis (PCA) is performed to reduce the correlation between the different features, and k-means clustering is performed in a five dimensional space obtained from the PCA analysis, in the same way as is done in Kang et al. (2015). The optimal number of clusters is calculated using a combination of statistical indicators as recommended in Charrad et al. (2014) and is found to be k = 3 for the SnoHATS dataset. Note that this is the same number that was found by Kang et al. (2015) for the FLOSSII dataset.

Table 3 shows the feature values for each cluster for the events extracted from the SnoHATS temperature data. The results of the clustering step are somewhat different than those of Kang et al. (2015). In the FLOSSII data as shown by their analyses, the clustered events exhibited clear differences in largest temperature change and wind direction change, with their cluster 3 corresponding to well-defined step-like structures. While our cluster 1 also gathers somewhat larger wind speed, more turbulent events and smoother structures and cluster 3 gathers sharper changes in temperature and wind direction, the differences between the events characteristics in the three clusters are not very clear.

The main physical characteristics of detected turbulent events are presented in Fig. 3. Inspection of the mean wind speed during events (Fig. 3a), of the vertical velocity variance σ_w (Fig. 3e) and of the bulk Richardson number (Fig. 3b) show that a weak distinction can be made between the events characteristics of cluster 1 on the one hand (with higher wind speeds and σ_w), and cluster 2 and 3 on the other hand (with



Figure 2: Heat flux MRD cospectra: mean over all 30-min periods within a flow regime, with standard deviation as errorbars. Left panels: SnoHATS. Right panels: FLOSSII. From top to bottom: Regime S1-S4 (left) and F1-F4 (right).

lower values of σ_w), based on the k-means clustering for the SnoHATS data set. The largest temperature change (3d) shows a distinction between events from cluster 3 with larger temperature changes on the one hand, and cluster 1 and 2 with smaller changes on the other hand. The wind direction characteristics (3c and 3f) shows that the events in cluster 2 and 3 are not easily distinguishable in their physical characteristics. As next we therefore preferably group the turbulent events based on the FEM-BV-VARX regimes.

The SnoHATS setup, with its 12 neighbouring sonic anemometers, additionally offers an opportunity to test how robust the TED method is when used on slightly different measurements (i.e. time series obtained from nearby instruments). Figure 4 shows the repartition of events and their k-means clustering affiliation for the time series provided from the different sonic anemometers, separately for the Regimes S1-S4. One can see that a large proportion of the events are detected by several instruments simultaneously. This figure also illustrates the qualitative difference in events occurring in the four flow regimes: although events from cluster 1 (red in the figure) occur relatively frequently in all regimes, events from cluster 2 (green) and 3 (blue) are much more represented in the submeso-influenced regimes 2 and 4. The differences in events characteristics represented in different flow regimes is discussed in the next section. The total numbers of events in each k-means cluster and FEM-BV-VARX flow regime are listed in Table 4.



Figure 3: Boxplots of main physical characteristics for events extracted from the SnoHATS data and sorted in k-means cluster 1, 2 and 3.

4.4 Characteristics of events in different flow regimes

Table 4 points to a clear preference for the events of cluster 1 in Regime S1 and S2 of the SnoHATS data, while Regime S3 and S4 have most events belonging to cluster 2 and 3. According to the information on the non-smoothness characteristics listed in Table 3, the preference goes for non-smooth events for increasing flow regime affiliation. Nevertheless because the k-means cluster affiliation of the turbulent events does not





Figure 4: Parts of the time series detected as events in the FEM-BV-VARX Regimes S1 to S4 (panels (b) to (e)) for all 12 sonic anemometers of the SnoHATS dataset (Fig. 1). In (b) to (e), results from different

sonic anemometers placed according to the sketch in panel (a) are shown on the y-axis, from sonic 1 (bottom) to sonic 12 (top). The colour indicate k-means cluster affiliation for the corresponding event (Red: cluster 1, green: cluster 2, blue: cluster 3). The x-axis corresponds to the number of points in the

time series.

point to very clear differences, we use only the event detection step of the TED method. Instead of the TED clustering step, we will show the physical characteristics of turbulent events in the FEM-BV-VARX flow regimes S1-S4 and F1-F4.

The time series within each regime are discontinuous and the TED method is applied to all continuous portions of the time series individually. Based on a Ri_b number classification, Kang et al. (2015) found that events occurred with similar frequencies for different stability ranges in the FLOSSII dataset. When comparing the frequency of occurrence of events in the flow regimes classified according to their scale interaction properties however, differences appear both in the FLOSSII data and in the SnoHATS data (Table 5). In the SnoHATS data, the events account for less than 10 % of the total time in the two regimes identified as little influenced by submeso motions and weakly stable (Regime S1: 7.48% and Regime S1: 9.21%), whereas events account for 14.17% of the total time in Regime S3 and 20.37% of the total time in Regime S4. These two regimes are characterised by a median value of R_{i_b} larger than 0.25 (see Table 2). Note that despite the high percentage of events in S4, the percentage of cases where the sub-mesoscale wind velocity is higher than wind speed is smaller in S4 than in S3 (Table 2). Similarly in FLOSSII, the most turbulent regime F1 exhibits the lowest frequency of events (13.72 %), while the most stable regimes (Regime F3 and F4) have the highest frequency of events (above 38%). A significant difference is however found in Regime F2 (also weakly stable) in which the frequency of events is large (35 %). It could be that submeso motions are well represented in this regime, but that the mean shear is strong enough for the turbulence not to be affected by the sub-mesoscale wind fluctuations. In fact Vickers and Mahrt (2007) showed that the cross-wind velocity

Number of events					
FEM-VARX regime	cluster 1	cluster 2	cluster 3	Σ	
S1	104	35	7	146	
S2	181	66	38	285	
$\mathbf{S3}$	138	102	115	355	
S4	277	339	391	1007	

Table 4: Number of events in each k-means cluster and each flow regime S1-S4 of the SnoHATS dataset.

variance due to sub-mesoscale motions systematically increased with increasing wind speed at the FLOSSII site. This was speculatively attributed to enhanced generation of topographically induced motions by a nearby ridge, and could partly explain the higher percentage of events for the higher wind speed regime F3, when compared to the more flat terrain features of SnoHATS. Figure 5 further shows the event duration in the four flow regimes in SnoHATS and FLOSSII. The event duration tends to increase with increasing regime affiliation number.

Table 5: Frequency of occurrence of the events for the regimes S1-S4 of SnoHATS and F1-F4 of FLOSSII.The numbers denote the percentage of the time detected as event within the total time of the

corresponding regime.

	SnoHATS/FLOSSII			
	S1/F1	S2/F2	S3/F3	S4/F4
events $[\%]$	7.8/13.7	9.2/35.0	14.1/43.4	20.3/38.2



Figure 5: Boxplot of the events duration shown for each FEM-BV-VARX regime separately and for each site.

The main physical characteristics of the events found in the different flow regimes are shown in Figure 6, discarding the TED event clustering step. As expected since Regime S1, S2, F1 and F2 correspond to weakly stable periods with little influence of sub-mesoscales on the turbulence, the bulk Richardson numbers during events are small, the wind speed is relatively high and the vertical velocity fluctuations are large in those regimes. The vertical velocity fluctuations decrease with increasing Regime affiliation number, with events in Regimes S4 and F4 associated to very little turbulent vertical velocity fluctuations.

The largest differences appear in the wind direction and temperature behaviour during events, in the most stable regimes. Events in Regimes S3 and F3 have a very large wind direction variability for both datasets. In Regimes S4 and F4, both characterised by a median Ri_b value larger than 0.6, the datasets differ markedly. The events of FLOSSII have a very large directional variability and large directional shifts.



Figure 6: Boxplot of the events physical properties, shown for events in each of the FEM-BV-VARX regimes for SnoHATS (S1-S4) and FLOSSII (F1-F4).

Such directional shifts were in fact shown to be common in the SBL under low wind speed by Mahrt (2010), based on the FLOSSII data. In this study, the strongest wind-direction shifts were shown to occur often with a sharp decrease of temperature (a cold microfront). This was also found by Lang et al. (2017) over a flat site in Australia. Moreover, Mahrt et al. (2012b) attributed an observed increase of wind directional shear at the FLOSSII site for increasing stratification to advection of cold air flow due to a cold pool forming upwind of the site. This is consistent with the events statistics for Regime F4, where the events are characterised by the largest wind directional shifts as well as the largest temperature changes. The events in the SnoHATS dataset however behave differently. The temperature changes are larger in Regime S4 than in Regime S3, but Regime S4 has the least wind direction variability. In fact analysis of the wind direction distribution during events in Regime S4 point to a preferred direction pointing straight towards the instruments. This direction corresponds to an opening at the end of the glacier, forming a funnel that probably induces a wind direction constrained by the topography (Fig. 1). The temperature changes in this regime are for a majority cold temperature changes (cold microfronts) as highlighted by the statistics in Fig. 6 panel (h), while events in other regimes have a strong tendency to correspond to warmer temperature changes. The cold microfronts events correspond to very little vertical velocity variance, and to rather small ratio of \hat{V}/V . This could indicate a regime where decreasing turbulence, surface cooling and increasing stratification could evolve together.

Regime S4 is thus characterised by submeso motions on scales significantly larger than the turbulent scales, that take a slow microfront signature with little wind direction variability. We hypothesise that advected air masses or density currents that tend to take a microfront structure, while enhancing shear locally, may only trigger little turbulence on small scales. Regime F4 has a similar scale signature and microfronts structures, but the site features are such that the microfronts also correspond to large shifts in the wind direction. Nevertheless, these events also trigger only little turbulent mixing. On the contrary, the wind-direction variability characteristics of events in regime S3, with its scale overlap, lead us to hypothesise that this regime encompasses wave-like phenomena that may break down to turbulence through a cascade of scales. The velocity of the submeso motions in this regime is often larger than the wind velocity.

4.5 Example of events and flow structures

The events were detected by the TED method based solely on the temperature time series, without considering information on the wind direction. In this section we want to explore the flow structures corresponding to selected events, taken as example in each identified flow regime. One example of event is shown for each flow regime of FLOSSII in Fig. 7, and for each flow regime of SnoHATS in Fig. 8. Examples are chosen that approximately match the median characteristics illustrated in 6. The timeseries of temperature T, wind speed U and σ_w for the events in F1 (Fig. 7a) and F2 (Fig. 7c) show that T and U tend to evolve in phase with a wavy pattern, but that σ_w does not follow the dynamics of T and U. This is in agreement with what was observed from the FEM-BV-VARX clustering analysis where no relationship was found between σ_w and the sub-mesoscale wind variability. The evolution of the horizontal wind components during the events is shown in Fig. 7b and Fig. 7d, where the corresponding evolution of the temperature is represented by the colours of the scatterplot. The black line in the figure is a smoothing of the time evolution of the wind vector, so as to smooth out the turbulence variability. In Fig. 7b and Fig. 7d, the wind vector evolves in a compact structure, and the temperature changes smoothly following the wind vector. In S1 and S2, the structure of the timeseries of T and U is similarly not followed by σ_w (Fig 8a and Fig 8c) as was observed by the FEM-BV-VARX clustering results. The time evolution of the wind vector shows rather a mixing process, with mixed temperature (Fig 8b and Fig 8d). In these weakly stable regimes, events are present but σ_w remains rather stationary during the events. The events of FLOSSII are more structured than at SnoHATS, which could be related to the differences in terrain complexity as discussed above.

In subsection 4.4 we pointed out that sharp temperature changes occur in the most stable flow regimes S3-S4 and F3-F4 (see Fig. 6d), being strongest in S4 and F4, while wind direction changes have a more site specific signature (see Fig. 6c). The wind direction variability during events is visible in the examples from F3 and F4, both highlighting a dispatched structure in the time evolution of the wind vector (Fig. 7f). The example of F4 has a sharp change of wind direction which is simultaneous to the sharp change of temperature (7h). This is the typical signature of a microfront which is commonly found with weak winds and thin stable boundary layers (Mahrt, 2010; Lang et al., 2017). For both examples, the dynamical evolution of σ_w is non-stationary, partly affected by the evolution of T and U (Fig. 7e and Fig. 7g). In the example from S3, the time evolution of the wind direction has a clear structure, but is more mixed with less sharp transitions than in the FLOSSII examples (Fig 8f). Increases in σ_w occur simultaneously to drops in the temperature (Fig. 8e), and the evolution of T and U is approximately in phase. In the example of S4 however, σ_w seems rather stationary again, and the drop in temperature and wind speed does not correspond to a marked increase of σ_w (Fig. 8g). The wind direction is oscillating (Fig. 8h. This example could correspond to a radiative cooling period, with wind meandering but a collapsed state of turbulence.

5 Conclusion

Flow regimes were classified in terms of interactions between submeso and turbulent scales of motion. In each flow regime, turbulent events were extracted using the TED method, and their statistical properties characterised. Regimes experiencing little scale interactions (S1, S2, F1, F2) are characterised by the shortest events, higher wind speeds, weak stability and fewer events.

Regimes experiencing more scale interactions correspond to higher stability, more numerous and longer events. In the most stable regimes that occur with weak winds, with a scale separation between turbulent and sub-mesoscales, the signature of events was found to strongly relate to local topographical characteristics. Events in these very weak wind conditions tend to exhibit strong temperature changes, with wind direction variability characteristics that depend on the local terrain features. The site differences exemplified here on two datasets render the derivation of parameterisations difficult. These flow regimes could be related to radiative cooling, advected air masses, density currents, and events thus tend to take a microfront structure with sharp temperature changes. Local shear enhancement due to the advected air masses results in turbulent mixing as identified by the FEM-BV-VARX method but the turbulent mixing occurs on very local, small scales. There is no cascade of scales but a separation of scales.

In regimes where the sub-mesoscale wind velocity is often larger than the wind velocity (S3, F3), or where the submeso and turbulent scales tend to overlap (S3), events are characterised by more variability in the wind direction but less sharp temperature changes. Events are associated with stronger vertical velocity fluctuations than in the very weak wind, strongly stable regimes. This could potentially be more related to wave-like phenomena that break down to turbulence through a cascade of scales.

The MRD analyses point to the randomness of sub-mesoscale contributions. The averaged contribution of the sub-mesoscales to the heat flux is negligible, however individual contributions become larger than turbulent contributions in strongly stable, weak wind regimes. The phenomena leading to the sub-mesoscale motions and associated fluxes are not resolved nor taken into account in numerical models, except through artificial enhanced mixing. The combination of flow regime detection to extract periods where submeso motions tend to dominate over turbulent transport and characterisation of events in each regime provides a way to define a stochastic process based on the statistical analyses.

The TED method is based on a principle of deviation from AR(1) processes to detect events that leave a trace in the timeseries which is significantly different from the typical trace of turbulence. This principle of deviation from AR(1) processes has also been used by Nevo et al. (2017) to investigate the hydrodynamical equilibrium properties of turbulence in different SBL flow regimes. The authors showed that intermittent or strongly stable regimes exhibit long memory effects in the turbulence dynamics. Submeso motions represent perturbations in the background turbulence, which are long-lived in strongly stable regimes. As a result, strongly stable, weak wind regimes need higher order stochastic models to represent turbulent mixing in a meaningful way.

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Figure 7: Visualisation of turbulent events for the FLOSSII dataset. One event as an example from each regime. The timeseries on the left show the temperature T (blue), the wind speed U (green) and the vertical velocity component w (black). The scatterplots show the temperature in colour, in the phase space of the horizontal velocity components. The black line is a spline smoothing of the time evolution of the



Figure 8: Visualisation of turbulent events for the SnoHATS dataset. One event as an example from each regime. The timeseries on the left show the temperature T (blue), the wind speed U (green) and the vertical velocity component w (black). The scatterplots show the temperature in colour, in the phase space of the horizontal velocity components. The black line is a spline smoothing of the time evolution of the wind vector.

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