Statistical and dynamical analyses of atmospheric blocking with an idealized point vortex model

By Mirjam Hirt\textsuperscript{1,2}, Lisa Schielick\textsuperscript{1}, Annette Müller\textsuperscript{1}, Peter Névir\textsuperscript{1}

\textsuperscript{1} Institut für Meteorologie, Freie Universität Berlin, Germany
\textsuperscript{2} Meteorologisches Institut, Ludwig-Maximilians-Universität, Munich, Germany

(Manuscript received xx xxxx xx; in final form xx xxxx xx)

ABSTRACT

We investigate a reduced point vortex model for a statistical and dynamical analysis of atmospheric blocking phenomena. Thereby, we consider high-over-low and omega blocking as equilibria of two and three point vortices. Based on fields of the kinematic vorticity number, two novel methods, the contour and the trapezoid method, are introduced in order to identify the vortices that form the blocking pattern as well as their local positions and circulation magnitudes. Using an instantaneous blocking index a total number of 347 blocking periods were identified in NCEP-NCAR Reanalysis data for the Euro-Atlantic region during the time period 1990-2012. This procedure provides the basis to corroborate the applicability of the point vortex model to atmospheric blocking in a statistical framework. The calculated translation speed of the point vortex systems associated with the atmospheric blocking appears to match the zonal mean velocity reasonably well. This model explains the stationary behaviour of blocking patterns. A comparison between the theoretical and a statistical model further reveals that the circulation of the blocking high follows the principles of the point vortex model to a large extent. However, the low-pressure-systems behave more variable. Moreover, the stability of point vortex equilibria is analysed regarding the relative distances by considering linear stability analysis and simulations. This reveals, that the point vortex blocking model corresponds to an unstable saddle point. Also, a possible transition between high-over-low and omega blocking situations is indicated. Furthermore, we take viscosity and a Brownian motion into account to simulate the influence of the smaller, subgrid-scale disturbances. As a result a clustering near the equilibrium state emerges indicating the persistence of the atmospheric blocking pattern.

Keywords: Atmospheric blocking, point vortices, kinematic vorticity number, stability analysis, instantaneous blocking index, circulation, vortex identification, vortex pattern recognition

1 Introduction

Blocking events are large-scale, quasi-stationary phenomena that persist from several days to weeks and block the jet stream and thus the westerly flow. In general, a blocked atmospheric flow field is characterized by a mid-tropospheric high pressure system that lies polewards of one or two lows. The pattern is called high-over-low in case of two vortices and Omega blocking in case of three vortices due to the Ω-shaped geopotential height isolines. Rex (1950) was one of the first who defined and studied blocking. Since then many theories have been developed to describe blocking: Charney and DeVore (1979) for example suggested that a metastable equilibrium state can be associated with blocking situations and Shutts (1983) proposed an eddy straining mechanism for the reinforcement and maintenance of blocking. Also many indices have evolved to detect blocked situations mostly in gridded model data. Well-known examples include those from Tihalidi and Molteni (1990) based on geopotential height gradients and from Pelly and Hoskins (2003) who introduced the PV-θ (Potential Vorticity - potential temperature) approach.

The persistent behaviour of blocking often causes extreme weather situations. An example of considerable impact is the Russian heatwave in summer 2010 which was accompanied by extreme rainfall in Pakistan (Galarneau Jr. et al., 2012). Despite their large and manifold impact on our society, numerical weather prediction models as well as climate models still need to be improved to produce adequate behaviour and appearance of blocking: blocking onsets frequently coincide with low forecast skill of numerical weather prediction models (Rodwell et al., 2013; Ferranti et al., 2015) and climate models of-
ten underestimate their frequency (Mitchell et al., 2017). These deficiencies are often ascribed to the still not sufficiently understood underlying dynamical mechanisms (e.g. Barnes et al., 2011; Yamazaki and Itoh, 2013; Luo et al., 2014; Pfahl et al., 2015; Kennedy et al., 2016).

Obukhov et al. (1984) were the first who considered blocking as a constellation of point vortices that on its own translates westward and becomes stationary within a counteracting horizontal westerly flow. Kuhlbrodt and Névir (2000) further considered a latitudinal dependent zonal mean flow resulting in a stable oscillation for dipole vortex constellations whose time scale corresponds to the oscillation of an exemplary high-over-low case. Further comparisons between case studies and point vortex systems also showed the transition from high-over-low to Omega blocking as well as the involvement of two neighbouring troughs in a four vortex framework (Kuhlbrodt and Névir, 2000). More recently Müller et al. (2015) demonstrated for two exemplary blocked weather situations that the magnitude of the translation velocity matches that of the zonal mean flow and thereby confirmed the stationary weather pattern. A similar view is presented by Altenhoff et al. (2008) regarding the blocking vortices as Potential Vorticity (PV) anomalies (instead of point vortices). These PV anomalies also counteract the ambient westerly flow leading to stationary conditions. This vortex perspective of blocking is complementary to other blocking theories, e.g. the development mechanism of blocking is often ascribed to Rossby wave breaking (Tyrlis and Hoskins, 2008). This mechanism enforces a transition from waves to vortices, supporting our vortex view.

Focusing on the stability of blocking, Faranda et al. (2015) proposed that blocking can be attributed to an unstable saddle point of the atmospheric dynamics. In the vicinity of this unstable saddle point clustering can occur manifesting in the persistence of blocking. This is fortified by Schubert and Lucarini (2016) showing that the atmospheric circulation is more unstable during blocking in comparison to unblocked flow.

In this study, we will focus on the following research questions:

(i) Can the applicability of the point vortex model to atmospheric blocking (Müller et al., 2015) be statistically corroborated, i.e. do atmospheric blocking behave similar to the point vortex model in general?

(ii) Which dynamical characteristics of blocking can be represented with the point vortex model?

(iii) How sensitive is the point vortex model to perturbations and what implications can be derived for its stability?

These research questions will be tackled in the following way: First, we will describe the theory of point vortices and how it can be applied to atmospheric blocking in Section 2. In order to give a more substantiated answer in a statistical framework, we will consider a large number of blocked weather situations instead of single examples. Therefore, we will present an automated, more objective methodology based on Müller et al. (2015) to detect blocking periods and to identify and characterize the vortices constituting the blocking in Section 3. Subsequently, the constituent blocking parameters are statistically investigated in Section 4. In Section 5 we will compare the theoretical point vortex model with a statistical model given by a linear multiple regression. We remark that with regard to atmospheric investigations reduced low-order dynamical models only rarely exist, allowing a comparison with statistical models based on reanalysis data sets. Furthermore, we will analyse the stability of blocked system by investigating the characteristics of the tripole relative equilibrium in Section 6. Finally, a summary and discussion will be given in Section 7.

2 The dynamical point vortex blocking model

The theory of point vortices is characterized by the interaction of discrete vortices under the idealized conditions of a two-dimensional, incompressible, inviscid flow. Mathematically it is represented by a system of coupled non-linear ordinary differential equations. Point vortices are determined by their circulation $\Gamma$, i.e. their strength, and their locations $\mathbf{r} = (x, y)$. The circulation is determined by the integral of the vorticity $\zeta$ over the vortex area $A$:

$$
\Gamma = \oint_A \zeta dA. \quad (1)
$$

The circulation can either be positive or negative corresponding to cyclonic or anticyclonic rotation. While the circulation is constant for each point vortex, the vorticity field is infinite at the point vortex locations and zero elsewhere. The equations of motion for $n$ point vortices are given by (Helmholtz, 1858):

$$
\frac{dx_i}{dt} = -\frac{1}{2\pi} \sum_{j=1, j\neq i}^{n} \frac{\Gamma_j(y_j - y_i)}{r_{ij}^2}, \quad (2)
$$

$$
\frac{dy_i}{dt} = \frac{1}{2\pi} \sum_{j=1, j\neq i}^{n} \frac{\Gamma_j(x_i - x_j)}{r_{ij}^2},
$$

where $l_{ij} = \sqrt{r_{ij}^2 - r_{ii}^2}$ denotes the distance between two point vortices $i$ and $j$. Thereby, each point vortex $i$ induces a velocity field that decreases with $l_{ij}^{-1}$. The superposition of the velocity fields induced by each point vortex then determines the motion of each vortex. Such point vortex systems conserve the horizontal Kelvin momenta, the angular momentum as well as the kinetic energy and therefore satisfy important physical characteristics of many fluid dynamical systems (see e.g. Müller et al., 2015). In general, point vortex systems rotate around their centre of circulation

$$
C = \sum_{i=1}^{n} \frac{\Gamma_i \mathbf{r}_i}{\sum_{i=1}^{n} \Gamma_i}, \quad (3)
$$

which is conserved due to the conservation of the Kelvin momenta. For systems with vanishing total circulation $\Gamma_{\text{total}} = \sum_{i=1}^{n} \Gamma_i = 0$ the centre of circulation moves to infinity. As a result, the system translates uniformly. An example of the motion of $n = 3$ point vortices with $\Gamma_{\text{total}} = 0$ arranged on an equilateral triangle is illustrated in Fig. 1.
Alternatively, point vortex systems can be described by their
intervortical distances \( l_{ij} \) as state variables, denoted as equa-
tions of relative motion (Gröbli, 1877; Aref, 1979; Newton,
2001):
\[
\frac{d l_{ij}^2}{d t} = \frac{2}{\pi} \sum_{k \neq i \neq j} \Gamma_k A_{ijk} \sigma_{ijk} \left( \frac{1}{l_{jk}^2} - \frac{1}{l_{ik}^2} \right), \quad \text{for } n \geq 3, \quad (4)
\]
where \( A_{ijk} \) describes the area and \( \sigma_{ijk} \) the orientation of the tri-
angle composed of three vortices \( i, j, k \). Thereby, \( \sigma \) is defined as
+1 for a counter-clockwise order of \( i, j, k \) and -1 for a clock-
wise order. Point vortex constellations that translate or rotate
uniformly by preserving their relative constellation are called
relative equilibria and correspond to fixed points in the frame-
work of the relative motion, i.e. the distances remain constant.

The point vortex constellation given in Fig. 1 corresponds to a
relative equilibrium due to the equalateral arrangement. More-
over, assuming \( \Gamma_{\text{total}} = 0 \), the point vortex system translates
uniformly. In case of \( \Gamma_{\text{total}} \neq 0 \) the point vortex constellation
rotates around its centre of circulation (3) but, as in the first case,
the intervortical distances remain constant. Both states are rel-
ative equilibria. For a more detailed overview on the theory of
point vortices we refer to Newton (2001); Aref (2007); Müller
et al. (2015).

The quasi-two-dimensional behaviour of atmospheric block-
ing allows for the representation of large-scale vortices by point
vortices as suggested by Obukhov et al. (1984). This reduces the
atmospheric flow field to a dynamical system described by ordi-
nary differential equations. Thereby, we identify the high pres-
sure system as anticyclonic point vortex and the low pressure
systems as cyclonic point vortices. The \( n = 2, 3 \) point vortex
systems representing the high-over-low and Omega blocking,
respectively, are illustrated in Fig. 2. In the high-over-low case
the circulation of the two vortices have the same absolute value
with opposite signs \( (\Gamma_1 = -\Gamma_2) \), whereas for the Omega case
the absolute value of the circulation of the anticyclonic vortex
\( (\Gamma_1) \) is equal to the sum of the circulation of the two cyclonic
vortices \( (\Gamma_2 = \Gamma_3 = -0.5 \Gamma_1) \), see also Fig. 1 for the Omega
case. Both cases are characterized by their vanishing total cir-
culation \( \Gamma_{\text{total}} = 0 \) which provoke the translation of the sys-
tems (see (3)). For uniform westward translation the vortices
are located on an equilateral triangle for the Omega case and on
the same longitude for the high-over-low case. Under these con-
ditions \( (\Gamma_{\text{total}} = 0) \), equilateral triangle) such point vortex con-
stellations correspond to relative equilibria and translate west-
wards with dipole velocity \( u_d = -u_d \hat{i} \) for the high-over-low
model and tripole velocity \( u_\Delta = -u_\Delta \hat{i} \) for the Omega case
(How, 2001):
\[
u_d = \frac{\vert \Gamma_1 \vert}{2 \pi l}, \quad \text{(5)}
\]
\[
u_\Delta = \sqrt{\frac{1}{2} (\Gamma_1^2 + \Gamma_2^2 + \Gamma_3^2)} \times \frac{1}{2 \pi l}, \quad \text{(6)}
\]
where \( l = l_{12} = l_{23} = l_{31} \) and \( \hat{i} \) is the unit vector pointing
to the east. For atmospheric blocking the zonal mean westerly
flow \( \bar{u} = \bar{u} \hat{i} \) counteracts this westward translation of the point
vortex system. As a result, the system can become stationary, if
the two velocities are of same magnitude:
\[
\bar{u} = \begin{cases} u_d & \text{for high-over-low blocking} \\ u_\Delta & \text{for omega blocking}. \end{cases} \quad \text{(7)}
\]

It is emphasized that the translation velocities \( u_d \) and \( u_\Delta \) cor-
respond to the theoretical translation of a corresponding point
vortex dipole/tripole. The actual, observable translation of a
non-stationary blocking system will be denoted as \( u_{\text{obs}} \).

3 Data and methods

3.1 Data and zonal mean flow

To analyse blocking systems, the NCEP-NCAR Reanalysis
Kalnay et al. (1996) is used with a horizontal resolution of
2.5° E x 2.5° N and a temporal resolution of 6 hours. We re-
stricted the analysis to blocking centred within 90° W - 90° E
(approximately the Euro-Atlantic sector) occurring in the years
1990-2012. For the analysis we used the fields at the 500 hPa-
level. The zonal mean flow \( \bar{u} \) is determined as the zonal average
of the global, zonal wind component within 20° - 80° N.

3.2 Identification of blocking periods

At first, the time periods of blocked atmospheric flows are
identified by using an Instantaneous Blocking Index (IBL)
which is implemented on the Freie Universität Berlin Evalu-
ation System (see freva, 2017; Richtel et al., 2015, for more
details). The blocking index is based on the 500 hPa geopo-
tential height gradient, similar to the detection method from
Tibaldi and Molteni (1990) combined with the approach of a
seasonal and longitudinal varying reference latitude which rep-
resents the position of the weather system activity (Pelly and
Hoskins, 2003; Barriopedro et al., 2010; Barnes et al., 2011).
Only those IBLs are considered as blocking periods that ex-
tend over at least 15° longitudes with one (or more) longitudes
blocked for a minimum of five days. Moreover, we determine an
IBL_{max} as the longitude that is blocked most frequently during
one blocking period. This IBL_{max} gives an approximate longi-
dudinal location of the blocking.

3.3 Identification of rotational flow using the kinematic
vorticity number

In a next step, we searched for prevalent rotational flow (i.e.
vortices) in the identified blocking periods. The search pro-
cedure is based on the dimensionless kinematic vorticity number
which was introduced by Truesdell (1953) as
\[
W_k^{(3D)} = \frac{\vert \Omega \vert}{\vert S \vert}, \quad \text{(8)}
\]
for three dimensions. Here, \( S \) and \( \Omega \) are the symmetric and anti-
symmetric tensors of the velocity gradient tensor \( \nabla v \). Recently,
the kinematic vorticity number was successfully applied to atmospheric data sets on two-dimensional surfaces by Schielicke et al. (2016). Explicitly, it reads:

\[ W_k^{(2D)} = \frac{\sqrt{\zeta^2}}{\sqrt{D_k^h + \text{Def} + \text{Def}^2}}, \]  

which can be evaluated at every point in the field and is used in this analysis. Here, \( \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \) is the vertical vorticity, \( D_k^h = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \) denotes the horizontal divergence, \( \text{Def} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \) defines the stretching deformation and \( \text{Def'} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \) denotes the shearing deformation. Hence, \( W_k^{(2D)} \) as well as \( W_k^{(3D)} \) characterise the relation between rotation, deformation and shearing of a flow (see Schielicke et al., 2016, for more details). We differentiate three cases:

\[ W_k < 1 : \ \text{deformation prevails over rotation} \]
\[ W_k = 1 : \ \text{pure shearing flow} \]
\[ W_k > 1 : \ \text{rotation predominates deformation} \]

As a result, rotational flow is identified as simply connected region of \( W_k > 1 \) which is used to define a vortex. For further analysis, we will only consider the vorticity field \( \zeta \) where \( W_k > 1 \), the other vorticity values are set to zero. This field will be called \( \zeta_{W_k>1} \). It represents a field of vortices that were cut out from the continuous flow field.

### 3.4 Vortex centre, circulations and intervortical distances

Under the assumption that we know the exact size of a vortex, we can define vortex properties such as the circulation and the vortex centre in the following way: The circulation \( \Gamma_i \) of vortex \( i \) is computed as the area weighted sum of vorticity as approximation to (1):

\[ \Gamma_i \approx \sum_m \Gamma_m = \sum_m \zeta_m a_m, \]  

where we sum over all \( n \) grid points that form vortex \( i \). \( \Gamma_m = \zeta_m a_m \) corresponds to the circulation of each grid point \( m \), that is approximated as the product of the vorticity \( \zeta_m \) and the area \( a_m \) of this grid point.

For each vortex \( i \) the location of its vortex centre \( C_i \) is calculated likewise to the centre of circulation of a point vortex system (3) as the circulation centre of all \( n \) grid points belonging to the area of the vortex \( i \):

\[ C_i = \frac{\sum_m \Gamma_m r_m}{\Gamma_i}, \]  

where \( r_m \) represents the grid point index of all grid points \( m \) belonging to the area of vortex \( i \). Although, this definition is similar to the definition of the circulation centre of a point vortex system, the latter is defined as centre of all \( n \) point vortices while the vortex centre is the circulation centre of a single extended vortex.

The intervortical distances \( l_{ij} \) between two vortices \( i \) and \( j \) are calculated as secants through the vortex centres.

### 3.5 Extracting vortex areas constituting the blocking

The most challenging part is to determine the areas of the vortices that constitute the blocking in an automated and objective way. In the following, we will introduce two methods, the contour and the trapezoid method, that have different approaches to determine these areas.

#### 3.5.1. Contour method for high-over-low and Omega blockings

Here, we will give a short overview of the contour method combining dynamical and statistical aspects; a detailed description can be found in the supplementary material (Section 1). A schematic diagram illustrating the method and an example are shown in Fig. 3. The contour method is based on the \( \zeta_{W_k>1} \) fields which are averaged over each blocking period. In these averaged \( \zeta_{W_k>1} \) fields, we identify stationary vortex structures as simply connected grid points with either statistically significantly positive or negative vorticity values. Significance is computed with a t-test (Wilks, 2005) based on a significance level \( \alpha \) which is initially set to \( \alpha_0 = 0.01 \). Coherent structures of such significant areas are identified by enclosing contours. These structures ideally represent isolated, persistent and stationary high (negative vorticity) or low (positive vorticity) pressure systems. In the following, the term contour refers to these values of significantly positive or negative, vorticity.

The high is determined by the contour with the smallest (negative) circulation that contains the IBL. Depending on their location and distance to the high, one or two of the nearest positive contours south of the high are chosen as the blocking lows (see the supplementary material for details). In case of one identified low in the averaged fields the whole blocking period is characterized as high-over-low, otherwise as Omega blocking. Yet sometimes the contours do not correspond to a single isolated vortex but enclose several connected vortex regions resulting in elongated contours. To avoid the selection of such elongated contours the \( \alpha \)-value is modified in case of unsuitable (e.g. too wide) high or low contours as illustrated in Fig. 3b. Whenever some variation in \( \alpha \) still fails to identify suitable contours, the whole blocking period is omitted.

Finally, we obtain a mask of stationary vorticity areas that represent the \( n = 2,3 \) vortices forming the blocking. The mask is derived on basis of the averaged fields. We will apply it to the 6-hourly fields in order to calculate the vortex centres, circulations and intervortical distances of the vortices constituting the blocking on a 6-hourly basis.

#### 3.5.2. Trapezoid method for Omega blockings

In contrast to the previously discussed method the basic concept of the trapezoid method is to determine the area of the blocking by a trapezoid that minimizes the total circulation as suggested by (Müller et al., 2015). Thereby, the upper part of the trapezoid corresponds to the high pressure system, while
the lower left and right parts correspond to the two low pressure systems (see Fig. 4). Therefore, it can only be applied to Omega blockings.

The trapezoid is determined for each single time step. This in order to determine the location and size of the trapezoid, the largest high pressure system is identified by the largest area enclosed by a negative $\zeta_{Wk} > 1$ contour ($\zeta_{Wk} > 1 < -10^{-8}$) within the blocked longitudes between $40^\circ - 85^\circ$N. This region was chosen, since it represents approximately the Jet region, where blocking develops. The contour needs to satisfy two constraints: (i) longitudes of the contour and the IBLs overlap by at least 25%, (ii) the ratio of the latitudes and longitudes covered by the contour is larger than 0.25. If no suitable contour for the high pressure system is found, the single time step is omitted. Otherwise the initial trapezoid is set according to Figure 4a, where the northern, north-western and north-eastern boundaries of the trapezoid are determined by the northern, western and eastern limits of the contour. The southern boundary of the trapezoid is initially set to be $30^\circ$ south of the averaged latitude of the high contour. The northern corners are always set to be $20^\circ$ longitudes smaller/larger than the corresponding northern values.

Inside this trapezoid three partly overlapping subregions are defined corresponding to the region of the blocking high and the two blocking lows (see Fig. 4a). Thereby, the southern boundary of the high’s subregion is given by the southern most latitude of the high contour. The subregions for the lows are bounded to the north by the averaged latitude of the high contour and separated by the mean longitude of the trapezoid. Only positive/negative vorticity values inside the subregions of the lows/high contribute to the circulation of the lows/high. Note, that the boundaries of the trapezoid might cut through vortices.

In order to minimize the total circulation $\Gamma_{total} = \Gamma_H + \Gamma_{Le} + \Gamma_{Lw}$ inside the trapezoid, small changes of the initial trapezoid are considered: Mainly the southern border is shifted up to $10^\circ$ north and south (in 2.5$^\circ$ intervals) since the more variable low pressure systems are more difficult to identify. This results in higher uncertainties for the southern border. Also the northern border is shifted up to $5^\circ$ to the north and the eastern and western boundaries also only up to $5^\circ$ to the east or the west. Only for very narrow initial trapezoids, i.e. when the upper width of the trapezoid is smaller than $40^\circ$ longitudes, shifts of up to $\pm 10^\circ$ are allowed. This yields a large number of different possible trapezoids. For each of the trapezoids the total circulation is calculated. The trapezoid that minimizes the total circulation is then chosen. An example comparing the initial and final trapezoid for a single time step is given in Fig. 4. Note, how the southern border of the final trapezoid (Fig. 4b) clearly deviates from the initial trapezoid (Fig. 4a) and how the final trapezoid adequately encloses the region of the blocking.

Finally, we determine the vortex centres, circulations and intervortical distances for each time step.

3.5.3. Differences between contour and trapezoid method

To summarize, in contrast to the contour method the trapezoid method is not able to distinguish between high-over-low or Omega blockings itself and is only applied to Omega blockings, that were previously identified by the contour method.

However, the trapezoid method allows for a translation of the blocking since vortex areas, i.e. the trapezoid, are determined for each single time step. In the contour method, the vortex areas are determined only once for the whole blocking period.

Furthermore, while the trapezoid method minimizes the total circulation to adopt the point vortex relative equilibrium condition ($\Gamma_{total} = 0$), there is no such constraint for the contour method. However, the latter rather displays complete, enclosed, albeit averaged vortex structures while the trapezoid method can cut through vortices in order to satisfy the minimization criterion.

3.6 Translation velocities

The translation velocity of the point vortex equilibria is computed according to (5) and (6). In case of the high-over-low blocking, (5) presumes both circulations to have the same absolute value. To account for deviations from this assumption, we will use the averaged absolute value of the circulations of the two vortices in the identified high-over-low cases.

In case of the Omega blocking, point vortex theory assumes that the vortices are arranged on an equilateral triangle of side length $l$. For the identified Omega blocking, we will use the average of the three intervortical distances for $l$ in (6). Minimum and maximum values of $u_A$ are calculated by using the maximum and minimum distance. We will consider these values as approximate error intervals.

4 Statistical analysis of the constituting blocking parameters based on NCEP data

In this section, we will present a climatology of the properties (composites, circulations, intervortical distance) of high-over-low and Omega blocking in the Euro-Atlantic sector for the years 1990-2012. The statistical analysis is based on the NCEP reanalysis data and the constituting vortices were identified with the methods described in Section 3.5.1. Furthermore, we will calculate the translation velocities and compare these to the zonal mean flow. Finally, we will shortly discuss the results and the methods.

4.1 Results

4.1.1. Composites and averaged blocking properties

The identification method (Section 3.2) found a total of 347 blocking periods during the time period 1990-2012 in the chosen area. With help of the contour method (Section 3.5.1.) we identified 106 of these blocking periods as high-over-low and
141 as Omega blocking periods. For the remaining 100 blocking periods, the method was not able to classify the pattern and these periods were disregarded. Both high-over-low and Omega cases were analysed by the contour method, but only the Omega cases were investigated by the trapezoid method. The composites for all Omega blocking and all high-over-lows are displayed in Fig. 5. Thereby the IBL$_{max}$ of each blocking period is relocated to $0^\circ E$ to enable a comparison between periods located at different longitudes. The flow in Fig. 5a is dominated by a high-over-low structure and the average strengths of the high and low are similar. The Omega structure for Omega blocking in Fig. 5b is less pronounced although differences between the high-over-low composite are visible. The composite for all identified Omega patterns shows a considerably weaker cyclonic vortex structure directly below the high than the high-over-low composite. While the latter shows almost vanishing vorticity south-east and south-west of the high the values in the Omega composite are clearly larger, consistent with the expected positions of the two lows.

The condition of vanishing total circulation is approximately satisfied for the trapezoid method ($\Gamma_{total}^{(trapez)} = 1 \times 10^7 m^2 s^{-1}$). In comparison, the cyclonic vortices dominate for the contour method ($\Gamma_{total}^{(contour)} = 3.5 \times 10^7 m^2 s^{-1}$). Furthermore, we observe that the contour method generally gives larger intervortical distances and smaller averaged circulations, especially for the high, compared to the trapezoid method (see Fig. 5b). This is further confirmed by a direct comparison of the two methods concerning all circulations averaged over each blocking period (see Fig. 6). This analysis shows that the contour method yields generally smaller values for the circulations of the highs than the trapezoid method. However, the circulations of the highs yield a high correlation while the circulations of the two lows are much less correlated.

4.1.2. Intervortical distances (6-hourly time steps):

The distances between the two vortices of the high-over-low blocking show a broad peak around 2200 km (see Fig. 7a). This is equal to a difference in latitudes of about $20^\circ$. While this distribution is approximately retained for the distances between the high and the lows of the Omega blocking, the distances between the two lows are significantly larger. This can be observed for both methods (see Fig. 7b,c). However, the contour method shows larger intervortical distances and wider, less regular distributions than the trapezoid method.

4.1.3. Circulations (6-hourly time steps):

For the high-over-low configurations (Fig. 7d) the maximum of the total circulation lies approximately at zero, suggesting that most high-over-low blockings consist of two equally strong vortices as the theory demands. For the Omega blocking, we observe that the circulations of the highs are generally larger for the trapezoid method than for the contour method. Regarding the lows this effect cannot be observed as clearly. The distributions of the total circulations $\sum \Gamma_i$ are centred symmetrically around zero for the trapezoid method (Fig. 7f). Because the minimized total circulation was chosen as constraint for the trapezoid selection, this is expected. For the contour method (Fig. 7e), the distribution of the total circulations also shows a maximum at approximately zero but the distribution is asymmetric in a way that more positive values are observed. This means that the two lows together tend to be stronger than the high for the contour method.

4.1.4. Comparing translation velocity and zonal mean flow

A central meteorological focus is the examination of the steady state of the blocked vortex configuration. Therefore, we compare the translation velocity magnitudes $u_u$ and $u_d$ with the zonal mean flow $\bar{u}$. Under the assumption of stationary blocking conditions, ideally, the absolute values of the translation velocity and zonal mean flow should be equal, i.e. the values of the corresponding scatter plots in Figure 8 should lie on the bisecting line for stationary blocking systems. For the Omega blockings analysed with the trapezoid method, the velocity values lie near the bisecting line (see Figure 8a). A significantly positive slope follows from a linear regression estimate with a correlation of 0.73. However, the linear regression differs considerably from the bisecting line: especially for large zonal mean velocities, $u_u$ is smaller than $\bar{u}$. This difference between $u_u$ and $\bar{u}$ remains also if $\bar{u}$ is calculated for other latitudinal bands, nonetheless, the observed slope of the linear regression estimate and its correlation still remain similar (not shown).

The contour method does not yield such a strong relationship between the two velocities, neither for the Omega nor the high-over-low blocking (Fig. 8b and 8c), since most $u_u$ and $u_d$ are smaller than $\bar{u}$ and the slope is more even. This could have several reasons and could be improved by a better handling of the identification of the vortices and thus a better estimation of the circulations and relative distances.

So far the blocking systems have been assumed to be stationary. However, many blocking translate slowly east- or westward and it is interesting to study the relation between this observed translation $u_{obs}$ and the difference $u_{diff}$ between the theoretical translation $u_u/u_d$ and the zonal mean flow $\bar{u}$. This difference is also visible in Fig. 8, which shows that the $u_u/u_d$ is generally smaller than $\bar{u}$. This suggests the possibility of more eastward propagating blocking systems. Examples (Omega blocking analysed with the trapezoid method) confirmed, that positive/negative $u_{diff}$ correspond to observed east-/westward translation $u_{obs}$ of the actual blocking system. Yet due to high variability of the blocking positions as analysed with the trapezoid method and the thereby arising difficulty in determining the translation $u_{obs}$ no statistically significant results could be obtained.

\[ \Gamma_{total}^{(trapez)} = 1 \times 10^7 m^2 s^{-1} \]
\[ \Gamma_{total}^{(contour)} = 3.5 \times 10^7 m^2 s^{-1} \]
4.2 Discussion of the statistical results and methods

A surprising result is the observed vorticity maximum south of the high in the Omega composite (Fig. 5b), as an ideal Omega pattern would suggest a gap in between the two lows. A possible explanation concerning the dynamics of this behaviour could be a larger variability of the locations of the lows in the Omega blocking cases. This means that the two lows are sometimes displaced more to the east or west of the high, which could result in these higher values directly south of the high. Possibly, there are more than 3 vortices involved or the real triangular arrangement of the vortices forming the Omega blocking could be a rotated Omega state such that the arrangement resembles a high-over-low with an additional second low located west or east of the high-over-low. This behaviour is an interesting aspect and might be related to the stability of point vortex equilibria which will be discussed in more detail in Section 6. It also indicates a possible transition between high-over-low and Omega configurations. Furthermore, this transition might mislead the contour method in assigning a pattern since its definition is quite strict: a blocking is either identified as high-over-low or Omega blocking, but not both. This might also be the reason why such a high number of blocking periods could not be assigned to either of the classes. Moreover, the contour method is not impeccable and more high-over-lows could have been mistaken as Omega pattern than the other way around.

One of the great challenges of atmospheric and fluid dynamics is the proper definition of the size of a vortex (e.g. Jeong and Hussain, 1995; Neu et al., 2013). Since ‘an accepted definition of a vortex is still lacking’ (Jeong and Hussain, 1995), we determined the areas of the blocking vortices with the contour and the trapezoid methods, i.e. two methods with different approaches. The contour method takes stationary persistent vortex structures over the whole blocking periods into account. Hence, it is rather related to the assumption that the blocking is formed by (the same) stationary vortices. In contrast, the trapezoid method selects the actual vortex areas at each time step with the constraint of minimum total circulation inside the trapezoidal pattern. This might lead to intersected lows. Furthermore, the blocking pattern can be formed by different individual vortices. Nevertheless, using two different methods has the advantage that we are able to evaluate the robustness of our results by comparing the outcomes of the two methods. Although, the contour method yields smaller values for the highs due to relatively small contours identified by the method, we observed that the circulations of the highs are well-correlated between both methods while the circulations of the two lows show a lower correlation (Figure 6). This suggests that the determination of the high is more reliable while the two lows are more difficult to capture, as they are more variable. Furthermore, the difficulty in capturing the areas of the low pressure systems also causes higher uncertainties in the position of the vortices.

An ideal point vortex Omega blocking requires an equilateral triangle. However, using reanalysis data sets we find that this is only approximately realized in the Omega blocking because the distance between the two lows is considerably larger than the distance between the high and the lows. Nonetheless, the relation between the calculated translation velocity $u_\Delta$ of the Omega blocking and the mean zonal flow $\bar{u}$ is a strong confirmation that the point vortex model is a reasonable description of atmospheric blocking. To further corroborate the applicability of the point vortex systems to blockings a statistical model of the blocking vortex system is considered and compared to the theoretical model in the following section.

5 Comparison of the theoretical and a statistical model of Omega blocking

The results derived in the previous section allows for a statistical model that can be compared to the analytic solution of the point vortex equation in a relative equilibrium. The tripole translation velocity $u_\Delta$ of the theoretical point vortex model given in (6) depends on the circulations and the intervortical distances. Thus the questions arise if one of these parameters contribute more to the relationship between the zonal mean flow $\bar{u}$ and $u_\Delta$ than others and how well the theoretical relationship of (6) fits to the observed one. We dealt with these questions with a multiple linear regression model (Wilks, 2005) representing the behaviour of Omega blocking.

5.1 Set-up of the theoretical and statistical models:

By considering only the behaviour near a reference point $a$, (6) can be approximated by a Taylor series expansion. As reference point we choose: $\pi = (\Gamma_H, \Gamma_{L_w}, \Gamma_{L_e}, l)$, where the bar above the variables denotes the average of the corresponding variable calculated from the methods. The indices stand for $H$: the high, $L_w$: the westerly low, $L_e$: the easterly low, and $l$ is the averaged intervortical distance. Then, the first order Taylor series for the tripole translation velocity reads:

$$ u_\Delta \approx u_\Delta(\pi) + \alpha_H(\Gamma_H - \bar{\Gamma}_H) + \alpha_{L_w}(\Gamma_{L_w} - \bar{\Gamma}_{L_w}) + \alpha_{L_e}(\Gamma_{L_e} - \bar{\Gamma}_{L_e}) + \alpha_l(l - \bar{l}), $$

(12)

where $\alpha_i$ with $i = (H, L_e, L_w, l, l_{H,L_e}, l_{H,L_w}, l_{L_e,L_w})$ are the corresponding derivatives at the reference point $\pi$. For example, $\alpha_H$ is given by:

$$ \alpha_H = \frac{\partial u_\Delta}{\partial \Gamma_H} \bigg|_{\pi} = \frac{\bar{\Gamma}_H}{4\pi l_0 \sqrt{0.5(\bar{\Gamma}_H^2 + \bar{\Gamma}_{L_w}^2 + \bar{\Gamma}_{L_e}^2)}}. $$

By using the averaged values at the reference point, the $\alpha_i$ become constants. In a next step, we assume $u_\Delta$ to have the same

\footnote{Note, $\bar{l}$ is considered as the average of the three distances, but also $\alpha$values are calculated using $l_{H,L_e}, l_{H,L_w}, l_{L_e,L_w}$.}
6.1 Stability considerations

In Section 4 we found that the distances between the three blocking vortices as computed with the contour and trapezoid method do not show an equilateral triangle. We will now analyse how such deviations from the equilateral triangle affect the point vortex system. In the following, the equations of motion for the relative distances \( d \) are applied to represent the equilateral triangle constellation as a fixed point in the phase space spanned by the three relative (intervortical) distances \( l_{ij} \) with \( i,j \in \{1,2,3\} \). An analysis considering the Lyapunov stability (see e.g. Strogatz, 2014) can then give information on the stability properties of the fixed point. A detailed derivation of this stability analysis can be found in the supplementary material (Section 2). A similar study has already been conducted by Synge (1949) (using trilinear coordinates) resulting in the following condition for stability:

\[
\Gamma_2 \Gamma_3 + \Gamma_1 \Gamma_2 + \Gamma_1 \Gamma_3 \geq 0.
\]

For the relations of the circulations according to the atmospheric blocking model, i.e. \( \Gamma_1 = -2 \Gamma_2, \Gamma_2 = \Gamma_3 > 0 \), the above stability criterion is not satisfied resulting in an unstable fixed point with \( \Gamma_2 \Gamma_3 + \Gamma_1 \Gamma_2 + \Gamma_1 \Gamma_3 = -3 \Gamma_2^2 < 0 \). Thus, within the vicinity of the fixed point deviations from the fixed point increase exponentially in time. More precisely the fixed point corresponds to a saddle point\(^2\) with one neutral, one unstable and one stable direction. This is illustrated in Figure 9, where three simulated trajectories are displayed in the vicinity of a fixed point (red cross). Each simulation is initialized at a perturbed state lying on the direction of an eigenvector. For the unstable case, the trajectory departs from the equilibrium constellation, whereas the stable trajectory converges towards the equilibrium. The neutral case corresponds to the uniform expansion of the equilateral triangle, which results again in a fixed point. However, trajectories, that do not start directly on the stable or neutral direction, are unstable. Therefore, the fixed point is unstable. See the supplementary material (Section 2), Synge (1949) or Tavantzis and Ting (1988) for further information.

6.1.1. Model set-up

To illustrate the non-linear behaviour of the initially unstable motion in the configuration space the positions of the point vortices have been simulated with perturbed equilateral triangles. In accordance with the results obtained from the NCEP statistics (Section 4, Fig. 7c,e), the circulations of the vortices were set to \( \left( \Gamma_H, \Gamma_{Le}, \Gamma_{Lw} \right) = (1.3, 0.65, 0.65) \cdot 10^8 \) s\(^{-1}\)m\(^2\) and the side length of the equilateral triangle was set to 2000 km. The integration is carried out by a Runge-Kutte-method of 4th order as implemented in Matlab (MATLAB, 2013). We used two different perturbed set-ups shown in Fig. 10a,b denoted as constellation 1. In the first simulation (Fig. 10a), we decreased the initial absolute value as \( \tilde{u} \). Then the above linearised theoretical equation (12) can be compared to the following model for a multiple linear regression:

\[
\bar{\tau} = \beta_0 + \beta_H \cdot \Gamma_H + \beta_{Lw} \cdot \Gamma_{Lw} + \beta_{Le} \cdot \Gamma_{Le} + \beta_{H \cdot Lw} \cdot l_{H \cdot Lw} + \beta_{H \cdot Le} \cdot l_{H \cdot Le} + \beta_{Le \cdot Lw} \cdot l_{Le \cdot Lw}
\]

The \( \beta \) values denote the corresponding regression estimates. In the case that the observed blocking, i.e. the determined values obtained from the contour and trapezoid methods, behave accordingly to the theoretical model, the \( \alpha \) values should coincide with the \( \beta \) values. Note, that we assumed that the blocking is stationary.

5.2 Results and Discussion:

For the trapezoid method the \( \alpha, \beta \) values are summarized in Table 1. Concerning the circulation of the blocked high pressure system \( \Gamma_H \) we have \( \alpha_H \approx \beta_H \) where a small p-value suggests significance. For the other parameters, the \( \alpha, \beta \) pairs do not match as well, but they are also less significant. Similar results can also be obtained for the contour method (not shown). Again, the theoretical value \((-2.7 \cdot 10^{-8} \) m\(^2\)s\(^{-1}\)) and the regression estimate \((-2.4 \pm 0.7 \cdot 10^{-8} \) m\(^2\)s\(^{-1}\)) for the circulation of the high fits adequately, while the others do not coincide as well. Although we neglect higher order terms in the Taylor series, the high pressure system (i.e. their circulations) behave in relation to the zonal mean flow in accordance with the simplified point vortex theory. This is remarkable, because it implies that the high pressure system of blocking situations can be described by this simplified point vortex theory to a certain extent.

In summary, this behaviour confirms with statistical significance what we have already inferred in Section 4 on a climatological basis: The circulation of the high pressure system is determined in a more reliable way whereas the other circulations and the distances are less trustworthy due to higher variability of the lows. We conclude, that the behaviour of the high is in accordance with the theoretical point vortex model.
distance between the two lows to 1800 km. In the second set-up (Fig. 10b), we increased the distance between the two lows to 3000 km (in accordance to Fig. 7c). In both cases, the initial triangle constellation is still isosceles and the distances between the high and the two lows remain \( l_{H Le} = l_{HLw} = 2000 \) km, roughly corresponding to their mean distance observed in Figure 7.

### 6.1.2 Results

Reducing the distance between the two lows leads to the following observations: The point vortices oscillate between the isosceles triangle constellations 1 and 4 and two other, collinear constellations 3 and 5 (Fig. 10a). It can be seen that the order of the vortices changes after the collinear constellations as the two lows switch their positions. This causes unstable eigenvectors to switch to stable ones (and reverse) leading to the attraction to the perturbed equilateral triangle, i.e. the isosceles triangle. As Constellation 2 moves away from the isosceles constellation towards the collinear constellation (i.e. the deviation from the equilateral triangle increases with time), it corresponds to an unstable point vortex constellations. Constellation 6 however converts to the isosceles constellation (i.e. the deviation from the equilibrium decreases) and thus represents a stable one. This behaviour can be viewed similar to the behaviour of real blocking events, where often a transition from high-over-low to Omega and reverse takes place. Moreover, variable locations of the lows can be explained, whereas the high pressure system is stationary over a longer time period.

An increase of the distance \( l_{LeLw} \) of the two lows in accordance with our statistics leads to an oscillating anticyclonic point vortex (see Fig. 10b), i.e. in the collinear state the high is located between both lows. Thereby, the distance between the high and the southern (northern) low increases (decreases). Ignoring the northern low, such a collinear state resembles a high-over-low configuration. In our case the time between the isosceles triangle constellation 1 and the collinear state 2 is about 6.2 days and a whole convulsion takes 12.4 days. The triangle configurations stay close to the isosceles pattern for about 3 days; e.g. constellation 2 in Fig. 10b is reached 1 day after the initialization (and a mirror constellation would be reached 1 day before configuration 1). Overall, the transition speed of the three point vortex system is smaller compared to set-up 1.

### 6.1.3 Discussion

Although persistent weather patterns are often denoted as stable weather situations in meteorological terms, the stability analysis of the corresponding point vortex system yields an unstable saddle point. This is also confirmed by Faranda et al. (2015) who indicate that blocking events correspond to an unstable saddle point (in the high dimensional phase space of the atmosphere) without considering any vortex models. Schubert and Lucarini (2016), using covariant Lyapunov vectors, also show that the atmospheric circulation is more unstable when the flow is blocked compared to non-blocked flow. This highlights that the concept of ‘stable’ (i.e. persistent) weather patterns does not necessarily correspond to stability in a dynamical systems view.

### 6.2 Clustering behaviour

Faranda et al. (2015) showed that clustering, i.e. an extraordinary long persistence near a point in phase space, can occur in the vicinity of unstable fixed points within chaotic attractors causing the persistence of blocking. These results motivated us to search for a clustering near the unstable fixed point of the point vortex blocking model to demonstrate the similarities of the point vortex blocking model with atmospheric blocking events.

#### 6.2.1 Model set-up

To eliminate the conservative character of our point vortex model friction was introduced according to Zhu and Cheng (2010) as Brownian motion. Thereby, (4) is complemented by a viscous and a noise term:

\[
\frac{dI^2_{ij}}{dt} = \frac{2}{\pi} \Gamma_{ij} A \sigma \left( \frac{1}{\Gamma_{jk}} - \frac{1}{\Gamma_{ik}} \right) + 8 \nu + \sqrt{8 \nu I_{ij}} \tilde{W}_{ij} \tag{13}
\]

where \( \nu \) represents the viscosity coefficient and \( W_{ij} \) the 1D Brownian motion for each \( i,j \). \( \tilde{W}_{ij} \) denotes the temporal derivative of \( W_{ij} \). Similar to Hasselmann (1976), who regarded weather as Brownian motion influencing the climate system, this noise can be considered as the impact of smaller scale phenomena on the positions of the larger scale blocking vortices. The modified point vortex system is regarded according to the Itô integral of stochastic differential equations as in Zhu and Cheng (2010) and numerical solutions are obtained using the Euler-Maruyama method. Thereby, \( \tilde{W}_{ij} = \mathcal{N}(0,\sigma^2)/\sqrt{dt} \) where \( \mathcal{N}(0,\sigma^2) \) denotes a normal distribution of zero mean and standard deviation \( \sigma \) (Higham, 2001).

We tested several (3721) initialisations (\( I_{LeLw}' = I_{LeLw}' \pm 30 \) km and \( I_{H Le}' = I_{H Le}' \pm 30 \) km in 1 km steps) with different initial intervortical distances in the vicinity of the mean isosceles triangle \( (I_{LeLw}', I_{H Le}', I_{H Lw}') = (3000,2000,2000) \) km that followed from the NCEP statistics. Accordingly, the circulations were set to \( (\Gamma_{H}, \Gamma_{Le}, \Gamma_{Lw}) = (1.3, 0.65, 0.65) \cdot 10^8 \) m\(^2\)/s. And the initial orientation of the triangle is \( \sigma = 1 \). The simulations were calculated with R (R Core Team, 2015) for time steps of 10 min over a total integration time of 4000 hours (\( \approx 166.7 \) days). The Brownian motion is modeled as normal distribution of zero mean and with standard deviation \( \sigma \) set to \( \sigma = 30 \) km. This \( \sigma \) value seems to be reasonable in comparison to the initial configuration based on the coarsely-resolved NCEP data (2.5°). For the viscosity we used the standard atmosphere kinematic viscosity at a height of 5500 m (\( \approx 500 \) hPa); \( \nu = 2.3 \cdot 10^{-5} \) m\(^2\)/s. We tested for clustering near
an equilateral triangle constellation. Thereby, clustering was defined as being close to an equilateral triangle constellation for at least 10 days over the whole integration time. The closeness was determined with help of the dimensionless distance

\[ \ell = \frac{\sqrt{\ell_{L,H,L,w}^2 + \ell_{H,L,e}^2 + \ell_{H,L,w}^2}}{\ell_{L,L,w} + \ell_{H,L,e} + \ell_{H,L,w}} \]  

(14)

in phase space. We required \( \ell < 0.03 \) for at least 10 days.

6.2.2. Results and Discussion

Although only for a fraction (\( \approx 1\% \)) of the tested set-ups, it was indeed possible to observe a clustering of the point vortex model near the equilateral triangle configuration during the integration times. An example is given in Fig. 11, where the system remains near the fixed point \( (l \approx 2000 \text{ km}) \) for about 15 days starting approximately at 105 days after the integration is initiated. Moreover, we notice that in the first period up to about 100 days the distance between one of the two lows and the high remains constant at about 1500 km and after the clustering the distance between the other low and the high is similarly stable while the other vortex moves more freely. This reminds of the high-overflow-low-dipole patterns with an additional vortex. However, the dipole might also rotate; hence, the high and low might change their positions. Nonetheless, it is an impressive result that even though we started far away from the equilateral triangle configuration the 3 point vortex system clusters close to the equilibrium state for such a long time period. Especially, since we used a realistic atmospheric conditions of the mid-troposphere for slightly viscous flow. This is a promising outcome that further confirms the applicability of the point vortex model to atmospheric blockings. However, further analyses (longer integration times, different set-ups, test for high-over-low-resembling behaviour) might be needed to give a more substantiated view of the point vortex clustering behaviour and its relation to the atmospheric blocking.

7 Conclusions

The focus of this paper is the corroborate of the applicability of the point vortex model to atmospheric blocking events. Two methods to identify and characterize blocking vortices in an automated way were proposed. The contour method selects the areas of the blocking vortices as contours of stationary vorticity and is able to distinguish between high-over-lows and Omega blockings. The trapezoid method after Müller et al. (2015) on the other hand is only applied to Omega blockings and adapts a trapezoid to fit the blocking vortices at each time step. Both methods evaluate a rather novel atmospheric field: the vorticity determined in the field of the dimensionless kinematic vorticity number \( W_k \) larger than 1 where the \( W_k > 1 \) criterion extracts the vortex structures embedded in the continuous flow field (see also Schielicke et al., 2016). From 347 blocking periods in total during 1990-2012, 106 were identified as high-over-lows and 141 as Omega blocking. A comparison of the two methods revealed that the high pressure systems were appropriately captured while the identification of the more variable lows is less reliable. The magnitudes of the circulations, distances and velocities are in accordance with the case studies of Müller et al. (2015). The condition of the vanishing total circulation is acceptably well satisfied, whereas clear deviations from the equilateral triangle are observed. However, the magnitude of the translation velocities \( u_{\Delta} \) and \( u_{\ell} \) of the point vortex tripole/dipole fits well with the zonal mean flow but the zonal mean flow is slightly stronger. Choosing different regions for the calculation of \( \bar{u} \) (e.g. smaller latitudinal bands or the latitudes within the selected trapezoid) only modifies the results slightly. Such differences could lead to non-stationary blocking systems and it was indeed observed that many blocking translate slowly.

Moreover, this allows us to compare the linearised analytic solution of the point vortex equilibrium with a statistical model. As a result of the multiple linear regression, we found that the circulation of the high pressure systems behaves in relation to the zonal mean flow according to the point vortex model. The circulations of the lows and the distances yield larger deviations between theory and statistics. It is commonly known that the persistent high pressure system is a major characteristic of blockings. Our analysis confirms that the high pressure system as anticyclonic vortex is dynamically relevant for the blocking phenomenon.

Another central point of this study was the analysis of the stability of the blocking, i.e. the response to perturbations from the equilateral triangle. A stability analyses was considered, finding that the equilateral triangle constellation (or the ideal point vortex blocking model) corresponds to an unstable saddle point in accordance with the findings from Faranda et al. (2015) and Schubert and Lucarini (2016). By considering the non-linear motion in the whole phase space (instead of only the local, linear behaviour near the fixed point), simulations showed an oscillatory behaviour of the lows in accordance with real blocking events. Thereby, a transition from Omega blocking to high-over-low is indicated. If the equilateral triangle is perturbed similar to the observed deviations, i.e. lows are further apart, the simulation reveals a more variable, oscillating anticyclonic vortex. This behaviour needs to be further studied in comparison to realistic atmospheric blocking behaviour, possibly using a higher number of point vortices. Furthermore, the clustering behaviour described in Faranda et al. (2015) can also be observed in the point vortex model concerning the relative distances when friction in terms of noise is included. This clustering may illustrate the persistent (‘stable’) behaviour of blocking as well as the difficulty in predicting the onset and offset of blocking. However, we notice that the reduced point vortex model does not include effects like divergence, baroclinicity, Rossby waves or the Earth’s rotation that also play a role in modifying the cancellation of the zonal mean flow and the theoretically calculated translation velocity from the point vortex blocking model. Other vortices, e.g. those embedded within the zonal mean flow, have
not been taken into account explicitly, only indirectly in terms of the averaged zonal mean flow.

To answer the research questions from the introduction we can conclude that atmospheric blockings, especially their high pressure systems, behave in many ways similar to the idealized point vortex blocking model. We have shown that not only the stationary behaviour of the blocking high can be modelled with point vortices, but also the instability and the consequently limited predictability due to clustering behaviour.

Acknowledgements

We like to thank Andy Richling for providing the Blocking Index data. Peter Névir and Annette Müller thank Deutsche Forschungsgemeinschaft for their support within the framework of CRC 1114 ‘Scaling Cascades in Complex Systems’, project A01. Lisa Schielicke was funded by the Helmholtz graduate school GeoSim.

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Table 1: Results of the multiple linear regression for Omega blocking as characterized with the trapezoid method. The \(\alpha\) values show the coefficients of the linearised point vortex equations and the \(\beta\) values denote the estimates from the linear regression. Small p-values indicate more significant regression estimates.
**Fig. 1.** Schematic illustration of the interaction of three point vortices arranged according to the atmospheric Omega pattern, where the circles indicate the direction and relative strength of rotation. The dotted arrows represent the influence of the other two vortices on the velocity of the corresponding point vortex. Their vector addition given by the solid lines represents the resulting velocity vector for the corresponding vortex. The anti-cyclonic vortex (red) is assumed to be twice as strong as the cyclonic vortices (blue), therefore the induced velocity field is stronger. This interaction can also be derived from Equations 2.

**Fig. 2.** (Left) Two exemplary blocking events, one resembling an Omega (top) and the other a high-over-low (bottom). Shown are the vorticity (coloured) and the geopotential height isolines (grey isolines in 8 dm intervals, bold line represents the 552 dm line) at 500 hPa. (Right) Illustration how the corresponding blocking can be realized in the point vortex model. Upper right figure by courtesy of Müller et al. (2015).
Fig. 3. Illustration of the contour method. (a) $\zeta_{Wk>1}$ (coloured) and geopotential height (grey isolines in 8 dm intervals, bold line represents the 552 dm line) fields for an exemplary blocking period (mean field). The blue and red contours enclose regions with significantly positive and negative vorticity ($\alpha = 0.5$). The solid black line qualitatively represents the number of timesteps, that the corresponding longitude was blocked during the blocking period. The dots mark the $IBL_{\text{max}}$. The identified blocking vortices are marked as bold (red and blue) contours and their centres are indicated by the circles with their circulation given in $10^7 m^2 s^{-1}$. (b) Flow chart of the contour method.
ANALYSING BLOCKING WITH AN IDEALIZED POINT VORTEX MODEL

Fig. 4. $\zeta_{W_k}>1$ and geopotential height fields (as in Figure 3a) for an exemplary time step to illustrate the trapezoid method. (a) The contour of the largest high pressure system defines the preliminary trapezoid: The minimum and maximum longitudes of the high contours define the northern boundaries of the trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in $10^7 m^2 s^{-1}$) and vortex centres marked as crosses.

Fig. 5. Composite of (a) all 106 high-over-low blockings and (b) all 141 Omega blockings that were identified from 347 blockings during 1990-2012. The mean positions and circulations (in $10^7 m^2 s^{-1}$) of the identified blocking vortices are marked for the contour method with circles and for the trapezoid method with crosses. The $\zeta_{W_k}>1$ and geopotential height fields are shown as in Figure 3a.
Fig. 6. Scatter plot of the circulations [$10^8 \text{m}^2\text{s}^{-1}$] (averaged for each blocking period) for the two methods. The dashed line shows the ideal case, the bisecting line. The regression line, the correlation coefficient and the $R^2$ value are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the western/eastern low.
Fig. 7. Histogram of the distances $l$ between the vortices (a-c) and the circulations $\Gamma$ (d-f) of the single time steps for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker.
Fig. 8. Scatter plot of the velocities $u_\Delta$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^\circ N$. The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance.

Fig. 9. Phase space of relative distances $l_{ij}$. The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary.
Fig. 10. Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The initial triangles (1) are disturbed from the relative equilibrium of the equilateral triangle of side length $l = 2000\,\text{km}$. The distance between the two lows is (a) decreased with $l_{LeLw} = 1800\,\text{km}$, (b) increased with $l_{LeLw} = 3000\,\text{km}$. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)\,\text{days}$; (b) for $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)\,\text{days}$. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the trilinear constellation and before the equilateral triangle constellation, as e.g constellation 6.

Fig. 11. Intervortical distances of the N=3 point vortex system of an exemplary simulation with friction as in Zhu and Cheng (2010). Initial set-up of the distances was $(l_{LeLw}, l_{HLe}, l_{HLw}) = (2981, 1995, 2000)\,\text{km}$. Random numbers were drawn from a normal Gaussian distribution of zero mean and standard deviation $sd = 30\,\text{km}$ using R function set.seed(12345) in order to estimate the Brownian motion. The other initial conditions are described in the text.
Table 1. Results of the multiple linear regression for Omega blocking as characterized with the trapezoid method. The $\alpha$ values show the coefficients of the linearised point vortex equations and the $\beta$ values denote the estimates from the linear regression. Small p-values indicate more significant regression estimates.

<table>
<thead>
<tr>
<th>predictor</th>
<th>theory ($\alpha$)</th>
<th>regression estimates ($\beta$)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_H$</td>
<td>$-3.7 \cdot 10^{-8} m^{-1}$</td>
<td>$-3.4 \pm 1.4 \cdot 10^{-8} m^{-1}$</td>
<td>0.02</td>
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<tr>
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<td>$2.2 \cdot 10^{-8} m^{-1}$</td>
<td>$-0.7 \pm 1.4 \cdot 10^{-8} m^{-1}$</td>
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<tr>
<td>$l$</td>
<td>$-3.2 \cdot 10^{-6} s^{-1}$</td>
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<td></td>
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<tr>
<td>$l_{Hel}$</td>
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<td>$2.0 \pm 1.2 \cdot 10^{-6} s^{-1}$</td>
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<tr>
<td>$l_{Helw}$</td>
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<td>$3.4 \pm 1.1 \cdot 10^{-6} s^{-1}$</td>
<td>0.75</td>
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<tr>
<td>$l_{Legw}$</td>
<td>$-2.1 \cdot 10^{-6} s^{-1}$</td>
<td>$-1.2 \pm 0.8 \cdot 10^{-6} s^{-1}$</td>
<td>0.10</td>
</tr>
</tbody>
</table>