# Statistical and dynamical analyses of atmospheric blocking with an idealized point vortex model

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## ABSTRACT

We investigate a reduced point vortex model for a statistical and dynamical analysis of atmospheric blocking phenomena. Thereby, we consider high-over-low and omega blocking as equilibria of two and three point vortices. Based on fields of the kinematic vorticity number, two novel methods, the contour and the trapezoid method, are introduced in order to identify the vortices that form the blocking pattern as well as their local positions and circulation magnitudes. Using an instantaneous blocking index a total number of 347 blocking periods were identified in NCEP-NCAR Reanalysis data for the Euro-Atlantic region during the time period 1990-2012. This procedure provides the basis to corroborate the applicability of the point vortex model to atmospheric blocking in a statistical framework. The calculated translation speed of the point vortex systems associated with the atmospheric blocking appears to match the zonal mean velocity reasonably well. This model explains the stationary behaviour of blocking patterns. A comparison between the theoretical and a statistical model further reveals that the circulation of the blocking high follows the principles of the point vortex model to a large extent. However, the low-pressure-systems behave more variable. Moreover, the stability of point vortex equilibria is analysed regarding the relative distances by considering linear stability analysis and simulations. This reveals, that the point vortex blocking model corresponds to an unstable saddle point. Also, a possible transition between high-overlow and omega blocking situations is indicated. Furthermore, we take viscosity and a Brownian motion into account to simulate the influence of the smaller, subgrid-scale disturbances. As a result a clustering near the equilibrium state emerges indicating the persistence of the atmospheric blocking pattern.

Keywords: Atmospheric blocking, point vortices, kinematic vorticity number, stability analysis, instantaneous blocking index, circulation, vortex identification, vortex pattern recognition

# 1 1 Introduction

<sup>2</sup>Blocking events are large-scale, quasi-stationary phenomena that persist from several days to weeks and block the jet stream and thus the westerly flow. In general, a blocked atmospheric flow field is characterized by a mid-tropospheric high pressure system that lies polewards of one or two lows. The pattern is called high-over-low in case of two vortices and Omega blocking in case of three vortices due to the  $\Omega$ -shaped geopotential height isolines. Rex (1950) was one of the first who defined and studied blocking. Since then many theories have been developed to describe blocking: Charney and DeVore (1979) for example suggested that a metastable equilibrium state can be

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<sup>13</sup> associated with blocking situations and Shutts (1983) proposed <sup>14</sup> an eddy straining mechanism for the reinforcement and main-<sup>15</sup> tenance of blocking. Also many indices have evolved to detect <sup>16</sup> blocked situations mostly in gridded model data. Well-known <sup>17</sup> examples include those from Tibaldi and Molteni (1990) based <sup>18</sup> on geopotential height gradients and from Pelly and Hoskins <sup>19</sup> (2003) who introduced the PV- $\theta$  (Potential Vorticity - potential <sup>20</sup> temperature) approach.

The persistent behaviour of blocking often causes extreme weather situations. An example of considerable impact is the Russian heatwave in summer 2010 which was accompanied by extreme rainfall in Pakistan (Galarneau Jr. et al., 2012). Despite their large and manifold impact on our society, numerical weather prediction models as well as climate models still need to be improved to produce adequate behaviour and appearance of blocking: blocking onsets frequently coincide with low forecast skill of numerical weather prediction models (Rodwell et al., 2013; Ferranti et al., 2015) and climate models of-

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31 ten underestimate their frequency (Mitchell et al., 2017). These st terize the vortices constituting the blocking in Section 3. Sub-32 33 34 2015; Kennedy et al., 2016). 35 Obukhov et al. (1984) were the first who considered block-36 ing as a constellation of point vortices that on its own trans-37 lates westward and becomes stationary within a counteracting

zonal westerly flow. Kuhlbrodt and Névir (2000) further con-30 sidered a latitudinal dependent zonal mean flow resulting in a 40 stable oscillation for dipole vortex constellations whose time 41 scale corresponds to the oscillation of an exemplary high-over-42 -low case. Further comparisons between case studies and point 43 vortex systems also showed the transition from high-over-low to 44 Omega blocking as well as the involvement of two neighbour-45 ing troughs in a four vortex framework (Kuhlbrodt and Névir, 46 2000). More recently Müller et al. (2015) demonstrated for 47 two exemplary blocked weather situations that the magnitude 48 of the translation velocity matches that of the zonal mean flow 49 and thereby confirmed the stationary weather pattern. A simi- 102 50 lar view is presented by Altenhoff et al. (2008) regarding the 51 blocking vortices as Potential Vorticity (PV) anomalies (instead 52 of point vortices). These PV anomalies also counteract the am-53 bient westerly flow leading to stationary conditions. This vor-54 tex perspective of blocking is complementary to other blocking 55 theories, e.g. the development mechanism of blocking is often 56 ascribed to Rossby wave breaking (Tyrlis and Hoskins, 2008). 57 This mechanism enforces a transition from waves to vortices, 58 supporting our vortex view. 59 Focusing on the stability of blocking, Faranda et al. (2015) 60 proposed that blocking can be attributed to an unstable saddle 61 point of the atmospheric dynamics. In the vicinity of this un-62

stable saddle point clustering can occur manifesting in the per-63 sistence of blocking. This is fortified by Schubert and Lucarini 64 (2016) showing that the atmospheric circulation is more unsta-65

ble during blocking in comparison to unblocked flow. 66

In this study, we will focus on the following research ques-67 tions: 68

(i) Can the applicability of the point vortex model to atmo-69 spheric blocking (Müller et al., 2015) be statistically corrobo-70 rated, i.e. do atmospheric blocking behave similar to the point 71 vortex model in general? 72

(ii) Which dynamical characteristics of blocking can be rep-73 resented with the point vortex model? 74

(iii) How sensitive is the point vortex model to perturbations 75 and what implications can be derived for its stability? 76

These research questions will be tackled in the following 122 77 way: First, we will describe the theory of point vortices and 78 how it can be applied to atmospheric blocking in Section 2. In 123 which is conserved due to the conservation of the Kelvin mo-79 order to give a more substantiated answer in a statistical frame- 124 menta. For systems with vanishing total circulation  $\Gamma_{total}$ 80 work, we will consider a large number of blocked weather situ-  $\sum_{i=1}^{n} \Gamma_i = 0$  the centre of circulation moves to infinity. As a 81 ations instead of single examples. Therefore, we will present an 126 result, the system translates uniformly. An example of the mo-82 automated, more objective methodology based on Müller et al. 127 tion of n = 3 point vortices with  $\Gamma_{total} = 0$  arranged on an 83 (2015) to detect blocking periods and to identify and charac- 128 equilateral triangle is illustrated in Fig. 1. 84

deficiencies are often ascribed to the still not sufficiently un- <sup>86</sup> sequently, the constituent blocking parameters are statistically derstood underlying dynamical mechanisms (e.g. Barnes et al., 87 investigated in Section 4. In Section 5 we will compare the 2011; Yamazaki and Itoh, 2013; Luo et al., 2014; Pfahl et al., <sup>88</sup> theoretical point vortex model with a statistical model given by <sup>89</sup> a linear multiple regression. We remark that with regard to at-<sup>90</sup> mospheric investigations reduced low-order dynamical models only rarely exist, allowing a comparison with statistical models 91 <sup>92</sup> based on reanalysis data sets. Furthermore, we will analyse the <sup>93</sup> stability of blocked system by investigating the characteristics <sup>94</sup> of the tripole relative equilibrium in Section 6. Finally, a sum-<sup>95</sup> mary and discussion will be given in Section 7.

# <sup>96</sup> 2 The dynamical point vortex blocking model

The theory of point vortices is characterized by the interac-97 tion of discrete vortices under the idealized conditions of a two-98 dimensional, incompressible, inviscid flow. Mathematically it is 99 represented by a system of coupled non-linear ordinary differ-100 ential equations. Point vortices are determined by their circula-101 tion  $\Gamma$ , i.e. their strength, and their locations  $\mathbf{r} = (x, y)$ . The circulation is determined by the integral of the vorticity  $\zeta$  over 103 the vortex area A: 104

$$\Gamma = \oint_{A} \zeta dA. \tag{1}$$

<sup>106</sup> The circulation can either be positive or negative correspond-107 ing to cyclonic or anticyclonic rotation. While the circulation is 108 constant for each point vortex, the vorticity field is infinite at <sup>109</sup> the point vortex locations and zero elsewhere. The equations of motion for n point vortices are given by (Helmholtz, 1858):

$$\frac{dx_i}{dt} = -\frac{1}{2\pi} \sum_{j=1, j \neq i}^n \frac{\Gamma_j(y_i - y_j)}{l_{ij}^2}, 
\frac{dy_i}{dt} = -\frac{1}{2\pi} \sum_{j=1, j \neq i}^n \frac{\Gamma_j(x_i - x_j)}{l_{ij}^2},$$
(2)

where  $l_{ij} = \sqrt{\mathbf{r}_i^2 - \mathbf{r}_j^2}$  denotes the distance between two point vortices i and j. Thereby, each point vortex i induces a velocity field that decreases with  $l_i^{-1}$ . The superposition of the velocity fields induced by each point vortex then determines the motion 115 of each vortex. Such point vortex systems conserve the hori-116 zontal Kelvin momenta, the angular momentum as well as the 117 kinetic energy and therefore satisfy important physical charac-119 teristics of many fluid dynamical systems (see e.g. Müller et al., 2015). In general, point vortex systems rotate around their cen-120 121 tre of circulation

$$C = \frac{\sum_{i}^{n} \Gamma_{i} \mathbf{r}_{i}}{\sum_{i}^{n} \Gamma_{i}},$$
(3)

(

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(5)

(6) 220

Alternatively, point vortex systems can be described by their 179 flow  $\bar{\mathbf{u}} = \bar{u}\mathbf{i}$  counteracts this westward translation of the point 129 intervortical distances  $l_{ij}$  as state variables, denoted as equa- 180 vortex system. As a result, the system can become stationary, if 130 tions of relative motion (Gröbli, 1877; Aref, 1979; Newton, 181 the two velocities are of same magnitude: 131 2001): 132

133 
$$\frac{dl_{ij}^2}{dt} = \frac{2}{\pi} \sum_{k \neq i \neq j}^n \Gamma_k A_{ijk} \sigma_{ijk} \left(\frac{1}{l_{jk}^2} - \frac{1}{l_{ik}^2}\right), \text{ for } n \ge 3, \quad (4)$$

134 135 +1 for a counter-clockwise order of i, j, k and -1 for a clock- 186 non-stationary blocking system will be denoted as  $u_{obs}$ . 136 wise order. Point vortex constellations that translate or rotate 137 uniformly by preserving their relative constellation are called 138 relative equilibria and correspond to fixed points in the frame-139 work of the relative motion, i.e. the distances remain constant. 188 3.1 Data and zonal mean flow 140 141 The point vortex constellation given in Fig. 1 corresponds to a relative equilibrium due to the equilateral arrangement. More-142 over, assuming  $\Gamma_{total} = 0$ , the point vortex system translates 143 uniformly. In case of  $\Gamma_{total} \neq 0$  the point vortex constellation 144 rotates around its centre of circulation (3) but, as in the first case, 145 the intervortical distances remain constant. Both states are rel-146 ative equilibria. For a more detailed overview on the theory of 147 point vortices we refer to Newton (2001); Aref (2007); Müller 148 et al. (2015). 149

The quasi-two-dimensional behaviour of atmospheric block-150 ing allows for the representation of large-scale vortices by point 197 151 vortices as suggested by Obukhov et al. (1984). This reduces the 152 atmospheric flow field to a dynamical system described by ordi-153 nary differential equations. Thereby, we identify the high pres-154 sure system as anticyclonic point vortex and the low pressure 155 systems as cyclonic point vortices. The n = 2, 3 point vortex 156 157 systems representing the high-over-low and Omega blocking, respectively, are illustrated in Fig. 2. In the high-over-low case 158 the circulations of the two vortices have the same absolute value 159 with opposite signs ( $\Gamma_1 = -\Gamma_2$ ), whereas for the Omega case 160 the absolute value of the circulation of the anticyclonic vortex 161  $(\Gamma_1)$  is equal to the sum of the circulation of the two cyclonic 162 vortices ( $\Gamma_2 = \Gamma_3 = -0.5 \Gamma_1$ , see also Fig. 1 for the Omega 163 case). Both cases are characterized by their vanishing total cir-164 culation  $\Gamma_{total} = 0$  which provoke the translation of the sys-165 tems (see (3)). For uniform westward translation the vortices 166 167 are located on an equilateral triangle for the Omega case and on the same longitude for the high-over-low case. Under these con-168 ditions ( $\Gamma_{total} = 0$ , equilateral triangle) such point vortex con-169 stellations correspond to relative equilibria and translate west-170 wards with *dipole velocity*  $\mathbf{u}_{\mathbf{d}} = -u_d \mathbf{i}$  for the high-over-low 171 model and *tripole velocity*  $\mathbf{u}_{\Delta} = -u_{\Delta}\mathbf{i}$  for the Omega case <sub>216</sub> 172 (Newton, 2001): 173

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$$u_{\Delta} = rac{2\pi l}{\sqrt{rac{1}{2}(\Gamma_1^2+\Gamma_2^2+\Gamma_3^2)}} {2\pi l},$$

 $u_{1} = \frac{|\Gamma_{1}|}{|\Gamma_{1}|}$ 

177 where  $l = l_{12} = l_{23} = l_{31}$  and **i** is the unit vector pointing 221 for three dimensions. Here, **S** and  $\Omega$  are the symmetric and anti-178 to the east. For atmospheric blocking the zonal mean westerly 222 symmetric tensors of the velocity gradient tensor  $\nabla \mathbf{v}$ . Recently,

$$\bar{u} = \begin{cases} u_d & \text{for high-over-low blocking} \\ u_\Delta & \text{for omega blocking.} \end{cases}$$
(7)

It is emphasized that the translation velocities  $u_d$  and  $u_{\Delta}$  corwhere  $A_{ijk}$  describes the area and  $\sigma_{ijk}$  the orientation of the tri- 184 respond to the theoretical translation of a corresponding point angle composed of three vortices i, j, k. Thereby,  $\sigma$  is defined as 185 vortex dipole/tripole. The actual, observable translation of a

# 187 **3 Data and methods**

To analyse blocking systems, the NCEP-NCAR Reanalysis (Kalnay et al., 1996) is used with a horizontal resolution of 190  $2.5^{\circ}E \times 2.5^{\circ}N$  and a temporal resolution of 6 hours. We restricted the analysis to blocking centred within  $90^{\circ}W - 90^{\circ}E$ 192 (approximately the Euro-Atlantic sector) occurring in the years 193 1990-2012. For the analysis we used the fields at the 500 hPalevel. The zonal mean flow  $\bar{u}$  is determined as the zonal average <sup>196</sup> of the global, zonal wind component within  $20^{\circ} - 80^{\circ}N$ .

## 3.2 Identification of blocking periods

At first, the time periods of blocked atmospheric flows are 100 identified by using an Instantaneous Blocking Index (IBL) which is implemented on the Freie Universität Berlin Evalu-200 ation System (see freva, 2017; Richling et al., 2015, for more 201 details). The blocking index is based on the 500 hPa geopo-202 tential height gradient, similar to the detection method from 203 204 Tibaldi and Molteni (1990) combined with the approach of a 205 seasonal and longitudinal varying reference latitude which rep-206 resents the position of the weather system activity (Pelly and 207 Hoskins, 2003; Barriopedro et al., 2010; Barnes et al., 2011). 208 Only those IBLs are considered as blocking periods that ex-209 tend over at least 15° longitudes with one (or more) longitudes blocked for a minimum of five days. Moreover, we determine an <sup>211</sup> IBL<sub>max</sub> as the longitude that is blocked most frequently during <sup>212</sup> one blocking period. This IBL<sub>max</sub> gives an approximate longi-213 tudinal location of the blocking.

### 3.3 Identification of rotational flow using the kinematic vorticity number 215

In a next step, we searched for prevalent rotational flow (i.e. vortices) in the identified blocking periods. The search proce-218 dure is based on the dimensionless kinematic vorticity number 219 which was introduced by Truesdell (1953) as

$$W_k^{(3D)} = \frac{\|\mathbf{\Omega}\|}{\|\mathbf{S}\|},\tag{8}$$

(9)

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<sup>223</sup> the kinematic vorticity number was successfully applied to at- <sup>269</sup> 3.5 Extracting vortex areas constituting the blocking mospheric data sets on two-dimensional surfaces by Schielicke 224 et al. (2016). Explicitly, it reads: 225

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$$W_k^{(2D)} = \frac{\sqrt{\zeta^2}}{\sqrt{D_h^2 + \mathrm{Def} + \mathrm{Def}'^2}},$$

which can be evaluated at every point in the field and is used 227 in this analysis. Here,  $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$  is the vertical vorticity, 228  $D_h = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$  denotes the horizontal divergence, Def =  $\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$  defines the stretching deformation and Def' =  $\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$ 229 230 denotes the shearing deformation. Hence,  $W_k^{(2D)}$  as well as 231  $W_{k}^{(3D)}$  characterize the relation between rotation, deformation 232 233 and shearing of a flow (see Schielicke et al., 2016, for more details). We differentiate three cases: 234

	$W_k < 1$	:	deformation prevails over rotation
235	$W_k = 1$	:	pure shearing flow
	$W_k > 1$	:	rotation predominates deformation

As a result, rotational flow is identified as simply connected 236 region of  $W_k > 1$  which is used to define a vortex. For further 237 analysis, we will only consider the vorticity field  $\zeta$  where  $W_k >$ 238 1, the other vorticity values are set to zero. This field will be 239 called  $\zeta_{W_h > 1}$ . It represents a field of vortices that were cut out 240 from the continuous flow field. 241

#### 3.4 Vortex centre, circulations and intervortical 242 distances 243

Under the assumption that we know the exact size of a vortex, 244 295 we can determine vortex properties such as the circulation and 245 the vortex centre in the following way: The *circulation*  $\Gamma_i$  of <sup>296</sup> 246 vortex i is computed as the area weighted sum of vorticity as <sup>297</sup> 247 approximation to (1): 248

<sup>249</sup> 
$$\Gamma_i \approx \sum_{m}^{n} \Gamma_m = \sum_{m}^{n} \zeta_m a_m,$$
 (10) <sup>300</sup>  
<sup>301</sup>

where we sum over all n grid points that form vortex i.  $\Gamma_m = {}^{302}$ 251  $\zeta_m a_m$  corresponds to the circulation of each grid point m, that 303 252 is approximated as the product of the vorticity  $\zeta_m$  and the area 253  $a_m$  of this grid point. 254

For each vortex i the location of its *vortex centre*  $C_i$  is cal-306 255 culated likewise to the centre of circulation of a point vortex 256 308 system (3) as the circulation centre of all n grid points belong-257 ing to the area of the vortex *i*: 258

$$\mathbf{C_i} = \frac{\sum_m^n \Gamma_m \mathbf{r}_m}{\Gamma_i},\tag{11}$$

where m represents the grid point index of all grid points n261 belonging to the area of vortex i. Although, this definition is 262 similar to the definition of the circulation centre of a point vor-263 tex systems, the latter is defined as centre of all n point vortices 314 264 while the vortex centre is the circulation centre of a single ex- 315 265 tended vortex. 266

267 are calculated as secants through the vortex centres. 268

The most challenging part is to determine the areas of the vortices that constitute the blocking in an automated and objective 271 way. In the following, we will introduce two methods, the contour and the trapezoid method, that have different approaches 273 to determine these areas. 274

# 275 3.5.1. Contour method for high-over-low and Omega blockings

Here, we will give a short overview of the contour method 277 combining dynamical and statistical aspects: a detailed descrip-278 tion can be found in the supplementary material (Section 1). A schematic diagram illustrating the method and an example are 280 shown in Fig. 3. The contour method is based on the  $\zeta_{W_L>1}$ 281 fields which are averaged over each blocking period. In these 282 averaged  $\zeta_{W_h>1}$  fields, we identify stationary vortex structures 283 as simply connected grid points with either statistically sig-284 nificantly positive or negative vorticity values. Significance is 285 computed with a t-test (Wilks, 2005) based on a significance 286 level  $\alpha$  which is initially set to  $\alpha_0 = 0.01$ . Coherent structures 287 of such significant areas are identified by enclosing contours. 288 These structures ideally represent isolated, persistent and sta-289 tionary high (negative vorticity) or low (positive vorticity) pres-290 sure systems. In the following, the term *contour* refers to these 291 values of significantly positive or negative, vorticity. 292

The high is determined by the contour with the smallest (neg-293 ative) circulation that contains the IBL $_{max}$ . Depending on their 294 location and distance to the high, one or two of the nearest positive contours south of the high are chosen as the blocking lows (see the supplementary material for details). In case of one identified low in the averaged fields the whole blocking period is 298 characterized as high-over-low, otherwise as Omega blocking. 200 Yet sometimes the contours do not correspond to a single iso-00 lated vortex but enclose several connected vortex regions resulting in elongated contours. To avoid the selection of such elongated contours the  $\alpha$ -value is modified in case of unsuitable (e.g. too wide) high or low contours as illustrated in Fig. 3b. When-304 ever some variation in  $\alpha$  still fails to identify suitable contours, 305 the whole blocking period is omitted.

Finally, we obtain a mask of stationary vorticity areas that represent the n = 2, 3 vortices forming the blocking. The mask is derived on basis of the averaged fields. We will apply it to the 309 6-hourly fields in order to calculate the vortex centres, circula-310 tions and intervortical distances of the vortices constituting the 312 blocking on a 6-hourly basis.

#### 3.5.2. Trapezoid method for Omega blockings 313

In contrast to the previous discussed method the basic concept of the trapezoid method is to determine the area of the <sup>316</sup> blocking by a trapezoid that minimizes the total circulation as The intervortical distances  $l_{ij}$  between two vortices i and j 317 suggested by (Müller et al., 2015). Thereby, the upper part of 318 the trapezoid corresponds to the high pressure system, while

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319 systems (see Fig. 4). Therefore, it can only be applied to Omega 320

321 blockings.

The trapezoid is determined for each single time step. In 322 order to determine the location and size of the trapezoid, the 323 largest high pressure system is identified by the largest area en-324 closed by a negative  $\zeta_{W_k>1}$  contour ( $\zeta_{W_k>1} < -10^{-8}$ ) within 325 the blocked longitudes between  $40^{\circ} - 85^{\circ}$ N. This region was 326 chosen, since it represents approximately the Jet region, where 327 blocking develops. The contour needs to satisfy two constraints: 328 (i) longitudes of the contour and the IBLs overlap by at least 329 25%, (ii) the ratio of the latitudes and longitudes covered by the 330 contour is larger than 0.25. If no suitable contour for the high 331 pressure system is found, the single time step is omitted. Oth-332 erwise the initial trapezoid is set according to Figure 4a, where 333 the northern, north-western and north-eastern boundaries of the 334 trapezoid are determined by the northern, western and eastern 335 limits of the contour. The southern boundary of the trapezoid is 336 initially set to be 30° south of the averaged latitude of the high 386 3.6 Translation velocities 337 contour. The southern corners are always set to be 20° longi-338 tudes smaller/larger than the corresponding northern values. 339 Inside this trapezoid three partly overlapping subregions are 340 defined corresponding to the region of the blocking high and the 341 two blocking lows (see Fig. 4a). Thereby, the southern boundary 342 of the high's subregion is given by the southern most latitude of 343 the high contour. The subregions for the lows are bounded to the 344 north by the averaged latitude of the high contour and separated 345 by the mean longitude of the trapezoid. Only positive/negative 346 vorticity values inside the subregions of the lows/high con-347 tribute to the circulation of the lows/high. Note, that the bound-348 aries of the trapezoid might cut through vortices. 349

In order to minimize the total circulation  $\Gamma_{total} = \Gamma_H +$ 350  $\Gamma_{Le} + \Gamma_{Lw}$  inside the trapezoid, small changes of the initial 351 trapezoid are considered: Mainly the southern border is shifted 352 up to  $10^{\circ}$  north and south (in  $2.5^{\circ}$  intervals) since the more vari-353 able low pressure systems are more difficult to identify. This 401 354 results in higher uncertainties for the southern border. Also the 355 northern border is shifted up to  $5^{\circ}$  to the north and the eastern 356 and western boundaries also only up to  $5^{\circ}$  to the east or the 357 west. Only for very narrow initial trapezoids, i.e. when the up-358 per width of the trapezoid is smaller than  $40^{\circ}$  longitudes, shifts 359 of up to  $\pm 10^{\circ}$  are allowed. This yields a large number of dif-360 ferent possible trapezoids. For each of the trapezoids the total 361 circulation is calculated. The trapezoid that minimizes the to-362 tal circulation is then chosen. An example comparing the initial 363 and final trapezoid for a single time step is given in Fig. 4. Note, 364 how the southern border of the final trapezoid (Fig. 4b) clearly 365 deviates from the initial trapezoid (Fig. 4a) and how the final 411 4.1 Results 366 trapezoid adequately encloses the region of the blocking. 367 Finally, we determine the vortex centres, circulations and in-368

tervortical distances for each time step. 369

## the lower left and right parts correspond to the two low pressure 370 3.5.3. Differences between contour and trapezoid method

To summarize, in contrast to the contour method the trape-371 zoid method is not able to distinguish between high-over-low 372 or Omega blockings itself and is only applied to Omega block-373 ings, that were previously identified by the contour method. 374 However, the trapezoid method allows for a translation of the 375 blocking since vortex areas, i.e. the trapezoid, are determined 376 for each single time step. In the contour method, the vortex ar-377 eas are determined only once for the whole blocking period. 378 Furthermore, while the trapezoid method minimizes the total 379 circulation to adopt the point vortex relative equilibrium condition ( $\Gamma_{total} = 0$ ), there is no such constraint for the contour 381 method. However, the latter rather displays complete, enclosed, 382 albeit averaged vortex structures while the trapezoid method can 383 <sup>384</sup> cut through vortices in order to satisfy the minimization criterion. 385

The translation velocity of the point vortex equilibria is computed according to (5) and (6). In case of the high-over-low 388 blocking, (5) presumes both circulations to have the same absolute value. To account for deviations from this assumption, 390 391 we will use the averaged absolute value of the circulations of the two vortices in the identified high-over-low cases. 392

In case of the Omega blocking, point vortex theory assumes 394 that the vortices are arranged on an equilateral triangle of side length l. For the identified Omega blocking, we will use the average of the three intervortical distances for l in (6). Minimum and maximum values of  $u_{\Delta}$  are calculated by using the maxi-397 mum and minimum distance. We will consider these values as approximate error intervals.

# 4 Statistical analysis of the constituting blocking parameters based on NCEP data

In this section, we will present a climatology of the properties (composites, circulations, intervortical distance) of high-403 over-low and Omega blocking in the Euro-Atlantic sector for the years 1990-2012. The statistical analysis is based on the 405 NCEP reanalysis data and the constituting vortices were identi-406 fied with the methods described in Section 3.5.1. Furthermore, 407 we will calculate the translation velocities and compare these to 408 the zonal mean flow. Finally, we will shortly discuss the results 409 410 and the methods.

#### 4.1.1. Composites and averaged blocking properties 412

The identification method (Section 3.2) found a total of 347 413 blocking periods during the time period 1990-2012 in the cho-414 sen area. With help of the contour method (Section 3.5.1.) we 415 416 identified 106 of these blocking periods as high-over-low and

141 as Omega blocking periods. For the remaining 100 block- 466 for the trapezoid method than for the contour method. Regard-417 418 419 420 421 422 423 424 425 426 are similar. The Omega structure for Omega blocking in Fig. 5b 476 high for the contour method. 427 is less pronounced although differences between the high-over-428 -low composite are visible. The composite for all identified 429 Omega patterns shows a considerably weaker cyclonic vortex 430 structure directly below the high than the high-over-low com-431 posite. While the latter shows almost vanishing vorticity south- 478 432 433 434 of the two lows. 435

The condition of vanishing total circulation is approximately 482 436 satisfied for the trapezoid method ( $\Gamma_{total}^{(trapez.)} = 1 \cdot 10^7 m^2 s^{-1}$ ). <sup>483</sup> ity and zonal mean flow should be equal, i.e. the values of the 437 In comparison, the cyclonic vortices dominate for the contour 484 438 method ( $\Gamma_{total}^{(contour)} = 3.5 \cdot 10^7 m^2 s^{-1}$ ). Furthermore, we ob- 485 ing line for stationary blocking systems. For the Omega block-439 serve that the contour method generally gives larger intervorti- 486 ings analysed with the trapezoid method, the velocity values lie 440 cal distances and smaller averaged circulations, especially for 487 near the bisecting line (see Figure 8a). A significantly positive 441 the high, compared to the trapezoid method (see Fig. 5b). This 488 slope follows from a linear regression estimate with a correla-442 is further confirmed by a direct comparison of the two methods 489 tion of 0.73. However, the linear regression differs considerably 443 concerning all circulations averaged over each blocking period 490 from the bisecting line: especially for large zonal mean veloci-444 (see Fig. 6). This analysis shows that the contour method yields 491 445 generally smaller values for the circulations of the highs than 492 446 the trapezoid method. However, the circulations of the highs 493 447 yield a high correlation while the circulations of the two lows 494 are much lesser correlated. 449

#### 4.1.2. Intervortical distances (6-hourly time steps): 450

The distances between the two vortices of the high-over-low 451 blocking show a broad peak around 2200 km (see Fig. 7a). This 452 is equal to a difference in latitudes of about  $20^{\circ}$ . While this 453 distribution is approximately retained for the distances between 454 the high and the lows of the Omega blocking, the distances be-455 tween the two lows are significantly larger. This can be observed 456 for both methods (see Fig. 7b,c). However, the contour method 457 shows larger intervortical distances and wider, less regular dis-458 tributions than the trapezoid method. 459

#### 4.1.3. Circulations (6-hourly time steps): 460

461 462 463 464 observe that the circulations of the highs are generally larger 516 sults could be obtained. 465

ing periods, the method was not able to classify the pattern and 467 ing the lows this effect cannot be observed as clearly. The disthese periods were disregarded. Both high-over-low and Omega  $_{468}$  tributions of the total circulations  $\sum \Gamma$  are centred symmetricases were analysed by the contour method, but only the Omega 469 cally around zero for the trapezoid method (Fig. 7f). Because cases were investigated by the trapezoid method. The compos- 470 the minimized total circulation was chosen as constraint for the ites for all Omega blocking and all high-over-lows are displayed 471 trapezoid selection, this is expected. For the contour method in Fig. 5. Thereby the IBL<sub>max</sub> of each blocking period is relo-472 (Fig. 7e), the distribution of the total circulations also shows cated to  $0^{\circ}E$  to enable a comparison between periods located at  $_{473}$  a maximum at approximately zero but the distribution is asymdifferent longitudes. The flow in Fig. 5a is dominated by a high- 474 metric in a way that more positive values are observed. This over-low structure and the average strengths of the high and low 475 means that the two lows together tend to be stronger than the

# 477 4.1.4. Comparing translation velocity and zonal mean flow

A central meteorological focus is the examination of the east and south-west of the high the values in the Omega com- 479 steady state of the blocked vortex configuration. Therefore, we posite are clearly larger, consistent with the expected positions  $_{480}$  compare the translation velocity magnitudes  $u_{\Delta}$  and  $u_d$  with the zonal mean flow  $\bar{u}$ . Under the assumption of stationary blocking conditions, ideally, the absolute values of the translation veloccorresponding scatter plots in Figure 8 should lie on the bisectties,  $u_{\Delta}$  is smaller than  $\bar{u}$ . This difference between  $u_{\Delta}$  and  $\bar{u}$  remains also if  $\bar{u}$  is calculated for other latitudinal bands, nonetheless, the observed slope of the linear regression estimate and its correlation still remain similar (not shown).

> The contour method does not yield such a strong relationship 495 between the two velocities, neither for the Omega nor the high-496 -over-low blocking (Fig. 8b and 8c), since most  $u_{\Delta}$  and  $u_d$  are 497 smaller than  $\bar{u}$  and the slope is more even. This could have sev-498 eral reasons and could be improved by a better handling of the 499 identification of the vortices and thus a better estimation of the 500 circulations and relative distances. 501

So far the blocking systems have been assumed to be sta-502 tionary. However, many blocking translate slowly east- or west-503 ward and it is interesting to study the relation between this 504 observed translation  $u_{obs}$  and the difference  $u_{diff}$  between 505 the theoretical translation  $u_{\Delta}/u_d$  and the zonal mean flow  $\bar{u}$ . This difference is also visible in Fig. 8, which shows that the 507  $u_{\Delta}/u_d$  is generally smaller than  $\bar{u}$ . This suggests the possi-508 bility of more eastward propagating blocking systems. Exam-509 ples (Omega blocking analysed with the trapezoid method) 510 sii confirmed, that positive/negative  $u_{diff}$  correspond to observed For the high-over-low configurations (Fig. 7d) the maximum  $_{512}$  east-/westward translation  $u_{obs}$  of the actual blocking system. of the total circulation lies approximately at zero, suggesting 513 Yet due to high variability of the blocking positions as analysed that most high-over-low blockings consist of two equally strong 514 with the trapezoid method and the thereby arising difficulty in vortices as the theory demands. For the Omega blocking, we sis determining the translation  $u_{obs}$  no statistically significant re-

#### 4.2 Discussion of the statistical results and methods 517

A surprising result is the observed vorticity maximum south 518 of the high in the Omega composite (Fig. 5b), as an ideal Omega 519 pattern would suggest a gap in between the two lows. A possible 520 explanation concerning the dynamics of this behaviour could 521 be a larger variability of the locations of the lows in the Omega 522 blocking cases. This means that the two lows are sometimes dis-523 placed more to the east or west of the high, which could result 524 in these higher values directly south of the high. Possibly, there 525 are more than 3 vortices involved or the real triangular arrange-526 ment of the vortices forming the Omega blocking could be a ro-523 tated Omega state such that the arrangement resembles a high-528 over-low with an additional second low located west or east of 529 the high-over-low. This behaviour is an interesting aspect and 581 530 might be related to the stability of point vortex equilibria which 582 531 will be discussed in more detail in Section 6. It also indicates a 583 532 possible transition between high-over-low and Omega configu- 584 533 rations. Furthermore, this transition might mislead the contour 585 534 method in assigning a pattern since its definition is quite strict: a 586 535 536 537 538 539 540 pattern than the other way around. 541

One of the great challenges of atmospheric and fluid dynam-542 ics is the proper definition of the size of a vortex (e.g. Jeong and 543 Hussain, 1995; Neu et al., 2013). Since 'an accepted definition 593 5.1 Set-up of the theoretical and statistical models: 544 of a vortex is still lacking' (Jeong and Hussain, 1995), we deter-545 mined the areas of the blocking vortices with the contour and the 546 trapezoid methods, i.e. two methods with different approaches. 547 The contour method takes stationary persistent vortex structures 548 over the whole blocking periods into account. Hence, it is rather 549 related to the assumption that the blocking is formed by (the 550 same) stationary vortices. In contrast, the trapezoid method se-551 lects the actual vortex areas at each time step with the constraint 552 of minimum total circulation inside the trapezoidal pattern. This 553 might lead to intersected lows. Furthermore, the blocking pat- 602 554 tern can be formed by different individual vortices. Neverthe-555 less, using two different methods has the advantage that we are 556 603 604 able to evaluate the robustness of our results by comparing the 557 outcomes of the two methods. Although, the contour method  $_{605}$  where  $\alpha_i$  with  $i = (H, Le, Lw, l, l_{HLe}, l_{HLw}, l_{LeLw})$  are the 558 yields smaller values for the highs due to relatively small con-  $_{606}$  corresponding derivatives at the reference point  $\overline{a}$ . For example, 559 tours identified by the method, we observed that the circulations  $_{607}$   $\alpha_H$  is given by: 560 of the highs are well-correlated between both methods while the 561 circulations of the two lows show a lower correlation (Figure 6). 608 562 This suggests that the determination of the high is more reliable 563 while the two lows are more difficult to capture, as they are 564 more variable. Furthermore, the difficulty in capturing the areas 565 of the low pressure systems also causes higher uncertainties in 566 the position of the vortices. 567 An ideal point vortex Omega blocking requires an equilateral 568

triangle. However, using reanalysis data sets we find that this

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570 is only approximately realized in the Omega blocking because 571 the distance between the two lows is considerably larger than the distance between the high and the lows. Nonetheless, the relation between the calculated translation velocity  $u_{\Delta}$  of the 573 574 Omega blocking and the mean zonal flow  $\bar{u}$  is a strong confirmation that the point vortex model is a reasonable description 575 of atmospheric blocking. To further corroborate the applicabil-576 ity of the point vortex systems to blockings a statistical model of the blocking vortex system is considered and compared to the 578 theoretical model in the following section. 579

# 5 Comparison of the theoretical and a statistical model of Omega blocking

The results derived in the previous section allows for a statistical model that can be compared to the analytic solution of the point vortex equation in a relative equilibrium. The tripole translation velocity  $u_{\Delta}$  of the theoretical point vortex model given in (6) depends on the circulations and the intervortical distances. blocking is either identified as high-over-low or Omega block- 587 Thus the questions arise if one of these parameters contribute ing, but not both. This might also be the reason why such a  $_{588}$  more to the relationship between the zonal mean flow  $\bar{u}$  and high number of blocking periods could not be assigned to either  $_{589}$   $u_{\Delta}$  than others and how well the theoretical relationship of (6) of the classes. Moreover, the contour method is not impeccable 590 fits to the observed one. We dealt with these questions with a and more high-over-lows could have been mistaken as Omega 591 multiple linear regression model (Wilks, 2005) representing the 592 behaviour of Omega blocking.

By considering only the behaviour near a reference point a, (6) can be approximated by a Taylor series expansion. As ref-595 erence point we choose:  $\overline{\mathbf{a}} = (\overline{\Gamma}_H, \overline{\Gamma}_{Lw}, \overline{\Gamma}_{Le}, \overline{l})$ , where the bar above the variables denotes the average of the corresponding 597 variable calculated from the methods. The indices stand for H: 598 the high, Lw: the westerly low, Le: the easterly low, and  $\overline{l}$  is 599 600 the averaged intervortical distance. Then, the first order Taylor series for the tripole translation velocity reads: 601

$$u_{\Delta} \approx u_{\Delta}(\overline{\mathbf{a}}) + \alpha_{H}(\Gamma_{H} - \overline{\Gamma}_{H}) + \alpha_{Lw}(\Gamma_{Lw} - \overline{\Gamma}_{Lw})$$
(12)  
+ $\alpha_{Le}(\Gamma_{Le} - \overline{\Gamma}_{Le}) + \alpha_{l}(l - \overline{l}),$ 

$$\alpha_H = \left. \frac{\partial u_\Delta}{\partial \Gamma_H} \right|_{\overline{\mathbf{a}}} = \frac{\overline{\Gamma}_H}{4\pi \overline{l} \sqrt{0.5(\overline{\Gamma}_H^2 + \overline{\Gamma}_{Lw}^2 + \overline{\Gamma}_{Le}^2)}}$$

<sup>610</sup> By using the averaged values at the reference point, the  $\alpha_i$  be-<sup>611</sup> come constants<sup>1</sup>. In a next step, we assume  $u_{\Delta}$  to have the same

<sup>&</sup>lt;sup>1</sup> Note,  $\overline{l}$  is considered as the average of the three distances, but also  $\alpha$ -values are calculated using  $\bar{l}_{HLe}, \bar{l}_{HLw}, \bar{l}_{LeLw}$ .

614 linear regression:

615 
$$\overline{u} = \beta_0 + \beta_H \cdot \Gamma_H + \beta_{Lw} \cdot \Gamma_{Lw} + \beta_{Le} \cdot \Gamma_{Le} + \beta_{l_H \ Lw} \cdot l_{H \ Lw} + \beta_{l_H \ Le} \cdot l_H \ Le + \beta_{l_{Le} \ Lw} \cdot l_{Le \ Lw}$$

The  $\beta$  values denote the corresponding regression estimates. In 665 the case that the observed blocking, i.e. the determined values 666 618 obtained from the contour and trapezoid methods, behave ac-619 cordingly to the theoretical model, the  $\alpha$  values should coincide 620 with the  $\beta$  values. Note, that we assumed that the blocking is 621 stationary. 622

#### 5.2 Results and Discussion: 623

For the trapezoid method the  $\alpha, \beta$  values are summarized in 624 Table 1. Concerning the circulation of the blocked high pres- 675 625 sure system  $\Gamma_H$  we have  $\alpha_H \approx \beta_H$  where a small p-value sug- 676 For the relations of the circulations according to the atmospheric 626 627 628 sults can also be obtained for the contour method (not shown). 629 Again, the theoretical value  $(-2.7 \cdot 10^{-8} m^2 s^{-1})$  and the re-630 gression estimate  $(-2.4 \pm 0.7 \cdot 10^{-8} m^2 s^{-1})$  for the circulation 631 632 633 634 635 fied point vortex theory. This is remarkable, because it implies 636 that the high pressure system of blocking situations can be de-637 scribed by this simplified point vortex theory to a certain extent. 638 In summary, this behaviour confirms with statistical signifi-689 639 640 641 termined in a more reliable way whereas the other circulations 642 and the distances are less trustworthy due to higher variability 643 of the lows. We conclude, that the behaviour of the high is in 694 Ting (1988) for further information. 644 accordance with the theoretical point vortex model. 645

#### 6 A stability analysis approach of blocked 646 systems 647

A remaining challenge in the context of large-scale atmo-648 spheric dynamics is the analysis of the stability of the block-649 ing phenomenon. For example, Rodwell et al. (2013) state that 650 weather prediction models often fail to capture the onset and 651 decay of blockings. So we will now examine (i) the stability of 652 blockings in terms of the Lyapunov stability of n = 3 point 653 vortex equilibria and by perturbing the side lengths of the equi-654 lateral triangle in accordance with the climatological results of 655 Section 4 and (ii) the clustering behaviour close to the relative 656 equilibrium state by modelling the influence of smaller, subgrid-657 scale disturbances as Brownian motion. 658

In Section 4 we found that the distances between the three blocking vortices as computed with the contour and trapezoid 661 method do not show an equilateral triangle. We will now analvse how such deviations from the equilateral triangle affect the 663 point vortex system. In the following, the equations of motion 664 for the relative distances (4) are applied to represent the equilateral triangle constellation as a fixed point in the phase space spanned by the three relative (intervortical) distances  $l_{ij}$  with 667  $i, j \in (1, 2, 3)$ . An analysis considering the Lyapunov stabil-668 ity (see e.g. Strogatz, 2014) can then give information on the stability properties of the fixed point. A detailed derivation of 670 this stability analysis can be found in the supplementary mate-671 rial (Section 2). A similar study has already been conducted by 672 Synge (1949) (using trilinear coordinates) resulting in the fol-673 lowing condition for stability:

$$\Gamma_2\Gamma_3 + \Gamma_1\Gamma_2 + \Gamma_1\Gamma_3 \ge 0.$$

gests significance. For the other parameters, the  $\alpha, \beta$  pairs do  $_{677}$  blocking model, i.e.  $\Gamma_1 = -2\Gamma_2, \Gamma_2 = \Gamma_3 > 0$ , the above stanot match as well, but they are also less significant. Similar re- 678 bility criterion is not satisfied resulting in an unstable fixed point with  $\Gamma_2\Gamma_3 + \Gamma_1\Gamma_2 + \Gamma_1\Gamma_3 = -3\Gamma_2^2 < 0$ . Thus, within the vicin-679 ity of the fixed point deviations from the fixed point increase ex-681 ponentially in time. More precisely the fixed point corresponds of the high fits adequately, while the others do not coincide as 682 to a saddle point<sup>2</sup> with one neutral, one unstable and one stable well. Although we neglect higher order terms in the Taylor se- 683 direction. This is illustrated in Figure 9, where three simulated ries, the high pressure systems (i.e. their circulations) behave in 684 trajectories are displayed in the vicinity of a fixed point (red relation to the zonal mean flow in accordance with the simpli- 685 cross). Each simulation is initialized at a perturbed state lying 686 on the direction of an eigenvector. For the unstable case, the tra-687 jectory departs from the equilibrium constellation, whereas the stable trajectory converges towards the equilibrium. The neutral case corresponds to the uniform expansion of the equilateral tricance what we have already inferred in Section 4 on a climato- 690 angle, which results again in a fixed point. However, trajectological basis: The circulation of the high pressure system is de- 691 ries, that do not start directly on the stable or neutral direction, <sup>692</sup> are unstable. Therefore, the fixed point is unstable. See the sup-602 plementary material (Section 2), Synge (1949) or Tavantzis and

# 695 6.1.1. Model set-up

To illustrate the non-linear behaviour of the initially unstable 696 motion in the configuration space the positions of the point vor-697 tices have been simulated with perturbed equilateral triangles. 698 In accordance with the results obtained from the NCEP statistics 699 (Section 4, Fig. 7c,e), the circulations of the vortices were set 700 to  $(\Gamma_H, \Gamma_{Le}, \Gamma_{Lw}) = (1.3, 0.65, 0.65) \cdot 10^8 \,\mathrm{m}^2/\mathrm{s}$  and the side 701 length of the equilateral triangle was set to 2000 km. The inte-702 gration is carried out by a Runge-Kutte-method of 4th order as 703 implemented in Matlab (MATLAB, 2013). We used two differ-704 ent perturbed set-ups shown in Fig. 10a,b denoted as constella-705 tion 1. In the first simulation (Fig. 10a), we decreased the initial 706

<sup>&</sup>lt;sup>2</sup> The saddle point arises from the existence of both stable/negative and unstable/positive eigenvalues.

710 the high and the two lows remain  $l_{HLe} = l_{HLw} = 2000 \,\mathrm{km}$  761 view. 711

roughly corresponding to their mean distance observed in Fig-712

ure 7. 713

#### 6.1.2. Results 714

715 716 717 718 719 two lows switch their positions. This causes unstable eigenvec- 770 events. 720 tors to switch to stable ones (and reverse) leading to the attrac-721 tion to the perturbed equilateral triangle, i.e. the isosceles tri-722 angle. As Constellation 2 moves away from the isosceles con-723 stellation towards the collinear constellation (i.e. the deviation 772 724 from the equilateral constellation increases with time), it corre- 773 model friction was introduced according to Zhu and Cheng 725 sponds to an unstable point vortex constellations. Constellation 774 (2010) as Brownian motion. Thereby, (4) is complemented by a 726 6 however converts to the isosceles constellation (i.e. the devia- 775 viscous and a noise term: 727 tion from the equilibrium decreases) and thus represents a stable 728 one. This behaviour can be viewed similar to the behaviour of 776 729 real blocking events, where often a transition from high-over- 777 730 -low to Omega and reverse takes place. Moreover, variable lo- 778 where  $\nu$  represents the viscosity coefficient and  $W_{ij}$  the 1D 731 cations of the lows can be explained, whereas the high pressure 779 Brownian motion for each  $l_{ij}$ .  $\dot{W}_{ij}$  denotes the temporal deriva-732 system is stationary over a longer time period. 733 734 cordance with our statistics leads to an oscillating anticyclonic 782 this noise can be considered as the impact of smaller scale 735 point vortex (see Fig. 10b), i.e. in the collinear state the high is 783 phenomena on the positions of the larger scale blocking vor-736 located between both lows. Thereby, the distance between the 784 tices. The modified point vortex system is regarded according 737 high and the southern (northern) low increases (decreases). Ig- 785 to the Itô integral of stochastic differential equations as in Zhu 738 noring the northern low, such a collinear state resembles a high-786 and Cheng (2010) and numerical solutions are obtained using 739 over-low configuration. In our case the time between the isosce- 787 the Euler-Maruyama method. Thereby,  $\dot{W}_{ii} = \mathcal{N}(0, sd)/\sqrt{dt}$ 740 741

days and a whole convulsion takes 12.4 days. The triangle con-789 standard deviation sd (Higham, 2001). 742 figurations stay close to the isosceles pattern for about 3 days: 790 743 744

ization (and a mirror constellation would be reached 1 day be-792 745 fore configuration 1). Overall, the translation speed of the three 793 746

point vortex system is smaller compared to set-up 1. 747

#### 6.1.3. Discussion 748

749 750 751 752 753 754 755 and Lucarini (2016), using covariant Lyapunov vectors, also  $_{805}$  ( $\approx 500$  hPa):  $\nu = 2.3 \cdot 10^{-5}$  m<sup>2</sup>/s. We tested for clustering near 756

(Fig. 10b), we increased the distance between the two lows to 758 flow is blocked compared to non-blocked flow. This highlights 3000 km (in accordance to Fig. 7c). In both cases, the initial tri- 759 that the concept of 'stable' (i.e. persistent) weather patterns does angle constellation is still isosceles and the distances between 760 not necessarily correspond to stability in a dynamical systems

## 762 6.2 Clustering behaviour

Faranda et al. (2015) showed that clustering, i.e. an extraor-763 <sup>764</sup> dinary long persistence near a point in phase space, can occur Reducing the distance between the two lows leads to the fol- 765 in the vicinity of unstable fixed points within chaotic attractors lowing observations: The point vortices oscillate between the 766 causing the persistence of blocking. These results motivated isosceles triangle constellations 1 and 4 and two other, collinear 767 us to search for a clustering near the unstable fixed point of constellations 3 and 5 (Fig. 10a). It can be seen that the order 768 the point vortex blocking model to demonstrate the similarities of the vortices changes after the collinear constellations as the 769 of the point vortex blocking model with atmospheric blocking

# 771 6.2.1. Model set-up

To eliminate the conservative character of our point vortex

$$\frac{dl_{ij}^2}{dt} = \frac{2}{\pi} \Gamma_k A\sigma \left( \frac{1}{l_{jk}^2} - \frac{1}{l_{ik}^2} \right) + 8\nu + \sqrt{8\nu} l_{ij} \dot{W}_{ij} \quad (13)$$

780 tive of  $W_{ii}$ . Similar to Hasselmann (1976), who regarded An increase of the distance  $l_{LeLw}$  of the two lows in ac- 781 weather as Brownian motion influencing the climate system, les triangle constellation 1 and the collinear state 2 is about 6.2 788 where  $\mathcal{N}(0, sd)$  denotes a normal distribution of zero mean and

We tested several (3721) initialisations  $(l'_{LeLw} = l_{LeLw} \pm$ e.g. constellation 2 in Fig. 10b is reached 1 day after the initial-  $\frac{1}{2}$  30 km and  $l'_{HLe} = l_{HLe} \pm 30$  km in 1 km steps) with different initial intervortical distances in the vicinity of the mean isosceles triangle  $(l_{LeLw}, l_{HLe}, l_{HLw}) = (3000, 2000, 2000) \text{ km}$ that followed from the NCEP statistics. Accordingly, the cir-794 culations were set to  $(\Gamma_H, \Gamma_{Le}, \Gamma_{Lw}) = (1.3, 0.65, 0.65)$ . 796  $10^8 \,\mathrm{m^2/s}$ . And the initial orientation of the triangle is  $\sigma =$ 797 +1. The simulations were calculated with R (R Core Team, Although persistent weather patterns are often denoted as 798 2015) for time steps of 10 min over a total integration time of stable weather situations in meteorological terms, the stability  $_{799}$  4000 hours ( $\approx 166.7$  days). The Brownian motion is modeled analysis of the corresponding point vortex system yields an un- 800 as normal distribution of zero mean and with standard deviastable saddle point. This is also confirmed by Faranda et al. 801 tion set to sd = 30 km. This sd-value seems to be reasonable (2015) who indicate that blocking events correspond to an un- 802 in comparison to the initial configuration based on the coarselystable saddle point (in the high dimensional phase space of the 803 resolved NCEP data (2.5°). For the viscosity we used the stanatmosphere) without considering any vortex models. Schubert 804 dard atmosphere kinematic viscosity at a height of 5500 m

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an equilateral triangle constellation. Thereby, clustering was de- 856 the two methods revealed that the high pressure systems were 806

807 at least 10 days over the whole integration time. The closeness 858 808

was determined with help of the dimensionless distance 809

$$\ell = \frac{\sqrt{l_{LeLw}^2 + l_{HLe}^2 + l_{HLw}^2}}{l_{LeLw} + l_{HLe} + l_{HLw}}$$

in phase space. We required  $\ell < 0.03$  for at least 10 days. 812

#### 6.2.2. Results and Discussion 813

Although only for a fraction ( $\approx 1\%$ ) of the tested set-ups, 814 it was indeed possible to observe a clustering of the point vor-815 tex model near the equilateral triangle configuration during the 816 integration times. An example is given in Fig. 11, where the 817 system remains near the fixed point ( $l \approx 2000 \,\mathrm{km}$ ) for about <sup>870</sup> 818 15 days starting approximately at 105 days after the integration 871 819 872 is initiated. Moreover, we notice that in the first period up to 820 about 100 days the distance between one of the two lows and 873 821 the high remains constant at about 1500 km and after the cluster-874 822 ing the distance between the other low and the high is similarly 875 823 stable while the other vortex moves more freely. This reminds 824 of the high-over-lowdipole patterns with an additional vortex. 877 825 However, the dipole might also rotate; hence, the high and low 878 826 879 might change their positions. Nonetheless, it is an impressive 827 result that even though we started far away from the equilat-828 eral triangle configuration the N=3 point vortex system clusters 829 882 close to the equilibrium state for such a long time period. Es-830 pecially, since we used a realistic atmospheric conditions of the 883 831 mid-troposphere for slightly viscous flow. This is a promising 884 832 outcome that further confirms the applicability of the point vor-833 tex model to atmospheric blockings. However, further analyses 834 longer integration times, different set-ups, test for high-over-835 low resembling behaviour) might be needed to give a more sub-836 889 stantiated view of the point vortex clustering behaviour and its 837 relation to the atmospheric blocking. 838

#### 7 Conclusions 839

840 ity of the point vortex model to atmospheric blocking events. 895 ulation reveals a more variable, oscillating anticyclonic vortex. 841 Two methods to identify and characterize blocking vortices 896 842 in an automated way were proposed. The contour method se- 897 alistic atmospheric blocking behaviour, possibly using a higher 843 lects the areas of the blocking vortices as contours of station- 898 number of point vortices. Furthermore, the clustering behaviour 844 ary vorticity and is able to distinguish between high-over-lows 899 described in Faranda et al. (2015) can also be observed in the 845 and Omega blockings. The trapezoid method after Müller et al. 900 point vortex model concerning the relative distances when fric-846 (2015) on the other hand is only applied to Omega blockings 901 tion in terms of noise is included. This clustering may illustrate 847 and adapts a trapezoid to fit the blocking vortices at each time 902 the persistent ('stable') behaviour of blocking as well as the 848 step. Both methods evaluate a rather novel atmospheric field: 903 difficulty in predicting the onset and offset of blocking. How-849 the vorticity determined in the field of the dimensionless kine- 904 ever, we notice that the reduced point vortex model does not 850 matic vorticity number  $W_k$  larger than 1 where the  $W_k > 1_{905}$  include effects like divergence, baroclinicity, Rossby waves or 851 criterion extracts the vortex structures embedded in the con- 906 the Earth's rotation that also play a role in modifying the can-852 tinuous flow field (see also Schielicke et al., 2016). From 347 907 cellation of the zonal mean flow and the theoretically calculated 853 blocking periods in total during 1990-2012, 106 were identified 908 translation velocity from the point vortex blocking model. Other 854 as high-over-lows and 141 as Omega blocking. A comparison of 909 vortices, e.g. those embedded within the zonal mean flow, have 855

fined as being close to an equilateral triangle constellation for 857 appropriately captured while the identification of the more variable lows is less reliable. The magnitudes of the circulations, distances and velocities are in accordance with the case stud-850 ies of Müller et al. (2015). The condition of the vanishing total 860 circulation is acceptably well satisfied, whereas clear deviations (14) 861 from the equilateral triangle are observed. However, the magni-862 tude of the translation velocities  $u_{\Delta}$  and  $u_d$  of the point vortex tripole/dipole fits well with the zonal mean flow but the zonal 864 mean flow is slightly stronger. Choosing different regions for 865 the calculation of  $\bar{u}$  (e.g. smaller latitudinal bands or the lat-866 itudes within the selected trapezoid) only modifies the results 867 slightly. Such differences could lead to non-stationary blocking 868 systems and it was indeed observed that many blocking trans-869 late slowly.

> Moreover, this allows us to compare the linearised analytic solution of the point vortex equilibrium with a statistical model. As a result of the multiple linear regression, we found that the circulation of the high pressure systems behaves in relation to the zonal mean flow according to the point vortex model. The circulations of the lows and the distances yield larger devia-876 tions between theory and statistics. It is commonly known that the persistent high pressure system is a major characteristic of blockings. Our analysis confirms that the high pressure system as anticyclonic vortex is dynamically relevant for the blocking phenomenon. 881

Another central point of this study was the analysis of the stability of the blocking, i.e. the response to perturbations from the equilateral triangle. A stability analyses was considered, finding that the equilateral triangle constellation (or the ideal point 885 vortex blocking model) corresponds to an unstable saddle point in accordance with the findings from Faranda et al. (2015) and Schubert and Lucarini (2016). By considering the non-linear motion in the whole phase space (instead of only the local, linear behaviour near the fixed point), simulations showed an oscillatory behaviour of the lows in accordance with real blocking events. Thereby, a transition from Omega blocking to high-over--low is indicated. If the equilateral triangle is perturbed similar The focus of this paper is the corroboration of the applicabil- 894 to the observed deviations, i.e. lows are further apart, the sim-This behaviour needs to be further studied in comparison to re-

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- not been taken into account explicitly, only indirectly in terms 961 H. Helmholtz, 1858. Über Integrale der hydrodynamischen Gleichun-910 of the averaged zonal mean flow. 911
- To answer the research questions from the introduction we 963 912
- can conclude that atmospheric blockings, especially their high <sup>964</sup> 913
- pressure systems, behave in many ways similar to the idealized 914
- point vortex blocking model. We have shown that not only the 915
- stationary behaviour of the blocking high can be modelled with 916
- point vortices, but also the instability and the consequently lim-917
- ited predictability due to clustering behaviour. 918

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# 1054 List of Figures

pattern, where the circles indicate the direction and relative strength of rotation. The dotted arrow represent the influence of the other two vortices on the velocity of the corresponding yortex. The anti-cyclenic vortex (red) is assumed to be twice as strong as the cyclenic vortices (blue), therefore the induced velocity field is stronger. This interaction can also be derived from Equations 2	1055	<b>Figure</b> 1 Schematic illustration of the interaction of three point vortices arranged according to the atmospheric Omega					
millione of the other two vortices on the velocity of the corresponding point vortex. Ther vector addition given by the solid lines represents the resulting velocity vector for the corresponding vortex. The anti-cyclonic vortex (red) is assumed to be twice as strong as the cyclonic vortices (hue), therefore the induced velocity field is stronger. This interaction can also be derived from Equations 2	1056	pattern, where the circles indicate the direction and relative strength of rotation. The dotted arrows represent the					
The solid lines represents the resulting velocity vector for the corresponding vortex. The anti-cyclence vortex (red) is assumed to be twice as strong as the cyclenic vortex (blue), therefore the induced velocity field is stronger. This interaction can also be derived from Equations 2. <b>Figure 2</b> (Left) Two exemplary blocking events, one resembling an Omega (top) and the other a high-over-low (bot- tom). Shown are the vorticity (coloured) and the geopotential height isolines (grey isolines in 8 dm intervals, bold line represents the 552 dm line) af 500 kPa. (Right) Illustration how the corresponding blocking can be realized in the point vortex model. Upper right figure by coursey of Miller et al. (2015). The solid black line qualita- tively represents the 552 dm line) fields for an exemplary blocking period (mean field). The blue and red contours enclose regions with significantly positive and negative vorticity ( $\alpha = 0.5$ ). The solid black line qualita- tively represents the number of timesteps, that the corresponding longitude was blocked during the blocking period. The dost mark the <i>IB Longer</i> . The identified blocking vortices are marked as bold (red and blue) contours and their centres are indicated by the circles with their circulation given in 10 <sup>7</sup> m <sup>2</sup> s <sup>-1</sup> . (b) How chart of the contour method. The dosts mark the <i>IB Longer</i> the high contours spatis defines the preliminary trangeroid: The minimum and maximum longitudes of the high contours define the northern boundaries of the trapezoid. The dished lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in 10 <sup>7</sup> m <sup>2</sup> s <sup>-1</sup> ) and votex centres marked as crosses	1057	influence of the other two vortices on the velocity of the corresponding point vortex. Their vector addition given by					
is assumed to be twice as strong as the cyclonic vortices (blue), therefore the induced velocity fields stronger. This interaction can also be derived from Equations 2	1058	the solid lines represents the resulting velocity vector for the corresponding vortex. The anti-cyclonic vortex (red)					
interaction can also be derived from Equations 2	1059	is assumed to be twice as strong as the cyclonic vortices (blue), therefore the induced velocity field is stronger. This					
Figure 2 (Left) Two exemplary blocking events, one resembling an Omega (top) and the other a high-over-low (betom). Shown are the vorticity (coloured) and the group tential height isolines (grey isolines in 8 dm intervals, bold line represents the 552 dm line) at 500 hPa. (Right) Illustration how the corresponding blocking can be realized in the point vortex model. Upper right figure 8 by courses of Muller et al. (2015)	1060	interaction can also be derived from Equations 2	15				
tom). Shown are the vorticity (coloured) and the geoptential height isolines (grey isolines in 8 dm intervals, hold ine represents the 552 dm line) at 500 hPa. (Right) Illustration how the corresponding blocking can be realized in the point vortex model. Upper right figure by courtesy of Muller et al. (2015)	1061	Figure 2 (Left) Two exemplary blocking events, one resembling an Omega (top) and the other a high-over-low (bot-					
line represents the 552 dm line) at 500 Phz. (Right) Illustration how the corresponding blocking can be realized in the point vortex model. Upper right figure by courtesy of Müller et al. (2015)	1062	tom). Shown are the vorticity (coloured) and the geopotential height isolines (grey isolines in 8 dm intervals, bold					
the point vortex model. Upper right figure by courtesy of Müller et al. (2015)	1063	line represents the 552 dm line) at 500 hPa. (Right) Illustration how the corresponding blocking can be realized in					
Figure 3 Illustration of the contour method. (a) $\zeta_{w_k>1}$ (coloured) and geopotential height (grey isolines in 8 dm intervals, bold line represents the 552 dm line) fields for an exemplary blocking period (mean field). The blue and red tively represents the number of timesteps, that the corresponding longitude was blocked during the blocking period. The dots mark the <i>IBL_mass</i> . The identified blocking vortices are marked as bold (red and blue) contours and their centres are indicated by the circles with their circulation given in $10^{7}m_s^{2-1}$ (b) Flow chart of the contour method. (a) The contour of the largest fligh pressure system defines the preliminary trapezoid: The minimum and maximum longitudes of the fligh contours define the northern boundaries of the trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in $10^7m_s^{2-1}$ ) and vortex centres marked as crosses	1064	the point vortex model. Upper right figure by courtesy of Müller et al. (2015)	15				
vals, bold line represents the 552 dm line) fields for an exemplary blocking period (mean field). The blue and red contours enclose regions with significantly positive and negative vorticity ( $\alpha = 0.5$ ). The solid black line qualita- tively represents the number of timesteps, that the corresponding longitude was blocked during the blocking period. The dots mark the <i>IB L</i> <sub>max</sub> . The identified blocking vortices are marked as bold (red and blue) contours and their centres are indicated by the circles with their circulation given in $10^7 m^2 s^{-1}$ . (b) Flow chard the contour method. 1 <b>Figure</b> 4 ( $\omega_{k_2}$ , 1 and geopotential height fields (as in Figure 3a) for an examplary time step to illustrate the trapezoid method. (a) The contour of the largest high pressure system defines the preliminary trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses	1065	<b>Figure</b> 3 Illustration of the contour method. (a) $\zeta_{W_1 > 1}$ (coloured) and geopotential height (grev isolines in 8 dm inter-					
contours enclose regions with significantly positive and negative vorticity ( $\alpha = 0.5$ ). The solid black line qualitatively represents the number of timesteps, that the corresponding longitude was blocked during the blocking period. The dots mark the <i>IBD</i> <sub>max</sub> . The identified blocking vortices are marked as bold (red and blue) contours and their centres are indicated by the circles with their circulation given in $10^7 m^2 s^{-1}$ . (b) Flow chart of the contour method. If figure 4 $(\kappa_w, >)$ and geopotential height fields (as in Figure 3a) for an examplary time step to illustrate the trapezoid method. (a) The contour of the largest high pressure system defines the preliminary trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses. (b) Adapted trapezoid with resulting circulations (in $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses. The $\zeta_{w_w, >1}$ and geopotential height fields are shown as in Figure 3a. (c) the trapezoid method with crosses. The $\zeta_{w_w, >1}$ and geopotential height fields are shown as in Figure 3a. (c) the two methods. The dashed line shows the ideal case, the bisecting line. The regression line, the correlation coefficient and the $R^2$ value are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the western/eastern low. (c) trapezoid method, and (c, f) rapezoid method, and (c, d) high-over-lows as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker. (c) $-0^{-1}$ Pase space of relative distances $l_{ij}$ . The fixed point is marked as star in corresponding column. Note that the grey trajectory lies on the neutral cignevector are displayed as points. The elagest dim he distances $l_{ij}$ . The fixed point is marked as the distance between the vortices instead of the average distance. (s) (s) (	1066	vals, bold line represents the 552 dm line) fields for an exemplary blocking period (mean field). The blue and red					
tively represents the number of timesteps, that the corresponding longitude was blocked during the blocking period. The dots mark the <i>IBLmax</i> . The identified blocking vortices are marked as bold (red and blue) contours and their centres are indicated by the circles with their circulation given in $10^7 m^2 s^{-1}$ . (b) Flow chart of the contour method. <b>Figure 4</b> $\zeta_{w_k,2}$ , and geopotential height fields (as in Figure 3a) for an examplary time step to illustrate the trapezoid method. (a) The contour of the largest high pressure system defines the preliminary trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations ( $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses	1067	contours enclose regions with significantly positive and negative vorticity ( $\alpha = 0.5$ ). The solid black line qualita-					
The dots mark the $IBL_{max}$ . The identified blocking vortices are marked as bold (red and blue) contours and their centres are indicated by the circles with their circulation given in $10^{17}m^2s^{-1}$ . (b) Flow chart of the contour method. 1 Figure 4 $\zeta_{w_k>1}$ and geopotential height fields (as in Figure 3a) for an examplary time step to illustrate the trapezoid method. (a) The contour of the largest high pressure system defines the preliminary trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses	1068	tively represents the number of timesteps, that the corresponding longitude was blocked during the blocking period.					
centres are indicated by the circles with their circulation given in $10^7 m^2 s^{-1}$ . (b) Flow chart of the contour method. I Figure 4 $\zeta_{w_k>1}$ and geopotential height fields (as in Figure 3a) for an examplary time step to illustrate the trapezoid method. (a) The contour of the largest high pressure system defines the preliminary trapezoid: The minimum and maximum longitudes of the high contours define the northern boundaries of the trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses	1069	The dots mark the $IBL_{max}$ . The identified blocking vortices are marked as bold (red and blue) contours and their					
<b>Figure 4</b> $\langle w_k \rangle_{k=1}$ and geopotential height fields (as in Figure 3a) for an examplary time step to illustrate the trapezoid method. (a) The contour of the largest high pressure system defines the preliminary trapezoid: The minimum and maximum longitudes of the high contours define the northern boundaries of the trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blockel longitudes. (b) Adapted trapezoid with resulting circulations (in $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses	1070	centres are indicated by the circles with their circulation given in $10^7 m^2 s^{-1}$ . (b) Flow chart of the contour method.	16				
right <b>4</b> <sup>−</sup> ( <i>w</i> <sub>k</sub> ), plane geopoential negatives (a) gives the system defines the preliminary trapezoid: The minimum and maximum longitudes of the high contours define the northern boundaries of the trapezoid. The distance in the largest high is used (local minimum) within the blocked longitudes. (b) Adapted the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted the latitudes, where the minimum vorticity is used (local minimum) within the block longitudes. (b) Adapted the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted the latitudes, where the minimum and maximum within the block longitudes. (b) Adapted the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted the latitudes, where the minimum and the distances (local minimum) within the blocked longitudes. (b) Adapted the latitudes, where the within the blocked longitudes. (b) Adapted the geopotential height fields are shown as in Figure 3a	1071	<b>Figure</b> $A = \frac{1}{2}$ (a) and geometerial height fields (as in Figure 3a) for an examplary time step to illustrate the transpoid					
The mean of the form of the high contours define the northern boundaries of the trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses	10/1	<b>Figure</b> 4 $(w_k)$ rand geopotential neight neight neight neight (as in Figure 5a) for an examplary time step to indistate the unperiod					
<b>Figure 8</b> Scatter plot of the velocities up the vortices (a-c) and the circulations $\Gamma$ (d-f) of the single time steps for Omega blocking a analysed with the contour method and (c,f) trapezoid method, and (a,d) high-over-low blockings is a nalysed with the contour method with circulations of the Orelation of the velocities when using the indicates the bisecting line. The regression line, the correlation coefficient and the $R^2$ value are shown for the high. The red points show the circulations of the Orelations $\Gamma$ (d-f) of the single time steps for Omega blocking a sanalysed with the contour method with circulations $\Gamma$ (d-f) of the single time steps for Omega blocking as analysed with the contour method and $u_d$ with the contal method with circulations $\Gamma$ (d-f) of the single time steps for Omega blocking as analysed with the (b,e) contain method and (c,f) trapezoid method, and (a,d) high-over-lows as analysed with the contour method and $u_d$ with the contain contained by averaged over $20 - 80^\circ N$ . The grey dashed line indicates the bisecting line, the bule line shows the line are very distributions, the colors accordingly appear darker 1 <b>Figure 8</b> Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\tilde{u}$ averaged over $20 - 80^\circ N$ . The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemptary trajectories are displayed as a points. The elashed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary	1072	maximum longitudes of the high contours define the porthern boundaries of the trapezoid. The dashed lines mark					
<b>Figure 5</b> Composite of (a) all 106 high-over-low blockings and (b) all 141 Omega blockings that were identified blocking vortices are marked for the contour method with circles and for the trapezoid method with crosses	10/3	the latitudes, where the minimum verticity is used (level minimum) within the blocked lengitudes. (b) Adapted					
<b>Figure 5</b> Composite of (a) all 106 high-over-low blockings and (b) all 141 Omega blockings that were identified from 347 blockings during 1990-2012. The mean positions and circulations (in $10^7 m^2 s^{-1}$ ) of the identified blocking vortices are marked for the contour method with circles and for the trapezoid method with crosses. The $\zeta_{w,k>1}$ and geopotential height fields are shown as in Figure 3a	1074	transported with resulting circulations (in $10^7 m^2 c^{-1}$ ) and vortex contrast marked as crosses	17				
Figure 5 Composite of (a) all 106 hgh-over-low blockings and (b) all 141 Omega blockings that were identified blocking 347 blockings during 1990-2012. The mean positions and circulations (in $10^7 m^2 s^{-1}$ ) of the identified blocking vortices are marked for the contour method with circles and for the trapezoid method with crosses. The $\zeta_{w_k>1}$ and geopotential height fields are shown as in Figure 3a	1075	trapezoid with resulting circulations (in 10 $m s$ ) and voltex centres marked as crosses	17				
347 blockings during 1990-2012. The mean positions and circulations ( $10^{1}m^{-5}s^{-1}$ ) of the identified blocking vortices are marked for the contour method with circles and for the trapezoid method with crosses. The $\zeta_{W_k>1}$ and geopotential height fields are shown as in Figure 3a	1076	Figure 5 Composite of (a) all 106 high-over-low blockings and (b) all 141 Omega blockings that were identified from $(1, 10^7, 2, -1)$ .					
workVortices are marked for the contour method with circles and for the trapezoid method with crosses. The $\{w_k\}$ -standgeopotential height fields are shown as in Figure 3a.11080Figure 6Scatter plot of the circulations $[10^8 m^2 s^{-1}]$ (averaged for each blocking period) for the two methods. The1081dashed line shows the ideal case, the bisecting line. The regression line, the correlation coefficient and the $R^2$ value1082are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the1083western/eastern low.1084Figure 7Histogram of the distances l between the vortices (a-c) and the circulations Γ (d-f) of the single time steps1084for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows1085as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker.1086Figure 8Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over 20 - 80° N.1087The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance.1089Figure 9Phase space of relative distances $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in co	1077	347 blockings during 1990-2012. The mean positions and circulations (in $10^{\circ}m^2s^{-1}$ ) of the identified blocking					
geopotential height fields are shown as in Figure 3a.1Figure 6Scatter plot of the circulations $[10^8 m^2 s^{-1}]$ (averaged for each blocking period) for the two methods. The dashed line shows the ideal case, the bisecting line. The regression line, the correlation coefficient and the $R^2$ value are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the western/castern low.11080Figure 7Histogram of the distances l between the vortices (a-c) and the circulations $\Gamma$ (d-f) of the single time steps for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker.110807Figure 8Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^\circ N$ . The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance.21090Figure 9Phase space of relative distances $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial position and is therefore stationary.21091Figure 10Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The initial position and is therefore stationary.21093Image disturbed from the relative equilibrium	1078	vortices are marked for the contour method with circles and for the trapezoid method with crosses. The $\zeta_{W_k>1}$ and					
Figure 6Scatter plot of the circulations $[10^8m^2s^{-1}]$ (averaged for each blocking period) for the two methods. The dashed line shows the ideal case, the bisecting line. The regression line, the correlation coefficient and the $R^2$ value are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the western/eastern low	1079	geopotential height fields are shown as in Figure 3a	17				
1081dashed line shows the ideal case, the bisecting line. The regression line, the correlation coefficient and the $R^2$ value are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the western/eastern low.11084Figure 7Histogram of the distances $l$ between the vortices (a-c) and the circulations $\Gamma$ (d-f) of the single time steps for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker.11086Figure 8Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^\circ N$ . The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance.210801Figure 9Phase space of relative distances $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial robingles (1) are disturbed from the relative equilibrium of the equilateral triangle of side length $l = 2000$ km. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for $($	1080	Figure 6 Scatter plot of the circulations $[10^8 m^2 s^{-1}]$ (averaged for each blocking period) for the two methods. The					
are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the western/eastern low	1081	dashed line shows the ideal case, the bisecting line. The regression line, the correlation coefficient and the $R^2$ value					
1083western/eastern low.11084Figure 7Histogram of the distances l between the vortices (a-c) and the circulations Γ (d-f) of the single time steps for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker.11085Figure 8Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^\circ N$ . The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance.21086Figure 9Phase space of relative distances $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary.21086Figure 10Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The initial triangles (1) are disturbed from the relative equilibrium of the equilateral triangle of side length $l = 2000$ km. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25$	1082	are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the					
Figure 7Histogram of the distances l between the vortices (a-c) and the circulations Γ (d-f) of the single time steps for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker.IFigure 8Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^\circ N$ . The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance.2Figure 9Phase space of relative distances $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary.2Figure 10Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The iolatance 	1083	western/eastern low.	18				
for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker 1 <b>Figure 8</b> Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^{\circ}N$ . The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance	1084	Figure 7 Histogram of the distances $l$ between the vortices (a-c) and the circulations $\Gamma$ (d-f) of the single time steps					
as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker 1 <b>Figure 8</b> Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^{\circ}N$ . The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance	1085	for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows					
Figure 8Scatter plot of the velocities $u_{\Delta}$ and $u_d$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^{\circ}N$ .1088The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance.1090 <b>Figure 9</b> Phase space of relative distances $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary.2 <b>Figure 10</b> Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The initial triangles (1) are disturbed from the relative equilibrium of the equilateral triangle of side length $l = 2000$ km. The distance between the two lows is (a) decreased with $l_{LeLw} = 1800$ km, (b) increased with $l_{LeLw} = 3000$ km. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation 2. Triangles, changed in the stable direction exist after the trilniear constellation and before the equilateral triangle constellation 6 2	1086	as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker.	19				
The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance	1087	<b>Figure 8</b> Scatter plot of the velocities $u_{\Delta}$ and $u_{d}$ with the zonal mean zonal velocity $\bar{u}$ averaged over $20 - 80^{\circ}N$ .					
<sup>1089</sup> Omegas) show the velocities when using the interact of the once the minimum and maximum distances between the vortices instead of the <sup>1089</sup> average distance	1088	The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for					
<b>Figure</b> 9 Phase space of relative distances $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary	1089	Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the					
<b>Figure</b> 9 Phase space of relative distances $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary	1000	average distance	20				
<b>Figure</b> 9 Finale space of relative distances $t_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary		Figure 0 Dessa space of relative distances l. The fixed point is marked as red gross and the three eigenvectors are	20				
<sup>1092</sup> displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemptary trajectories are displayed as <sup>1093</sup> points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star <sup>1094</sup> in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is <sup>1095</sup> therefore stationary	1091	Figure 9 Flass space of feative distances $l_{ij}$ . The fixed point is marked as fed closs and the three eigenvectors are displayed as green (stable), blue (unstable) and green (neutral) lines. Three examples trainestaries are displayed as					
points. The elapsed time between two consecutive points corresponds to 8n. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary	1092	displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemptary trajectories are displayed as					
in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary	1093	points. The elapsed time between two consecutive points corresponds to 8n. The initial condition is marked as star					
<b>Figure 10</b> Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The initial triangles (1) are disturbed from the relative equilibrium of the equilateral triangle of side length $l = 2000$ km. The distance between the two lows is (a) decreased with $l_{LeLw} = 1800$ km, (b) increased with $l_{LeLw} = 3000$ km. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the trilinear constellation and before the equilateral triangle constellation 6	1094	in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is	20				
<b>Figure 10</b> Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The initial triangles (1) are disturbed from the relative equilibrium of the equilateral triangle of side length $l = 2000$ km. The distance between the two lows is (a) decreased with $l_{LeLw} = 1800$ km, (b) increased with $l_{LeLw} = 3000$ km. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the trilinear constellation and before the equilateral triangle constellation 6	1095	therefore stationary	20				
(1) are disturbed from the relative equilibrium of the equilateral triangle of side length $l = 2000$ km. The distance between the two lows is (a) decreased with $l_{LeLw} = 1800$ km, (b) increased with $l_{LeLw} = 3000$ km. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the trilinear constellation and before the equilateral triangle constellation, as e.g constellation 6	1096	<b>Figure</b> 10 Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The initial triangles					
between the two lows is (a) decreased with $l_{LeLw} = 1800$ km, (b) increased with $l_{LeLw} = 3000$ km. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the trilinear constellation and before the equilateral triangle constellation, as e.g constellation 6	1097	(1) are disturbed from the relative equilibrium of the equilateral triangle of side length $l = 2000$ km. The distance					
lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the trilinear constellation and before the equilateral triangle constellation, as e.g constellation 6	1098	between the two lows is (a) decreased with $l_{LeLw} = 1800$ km, (b) increased with $l_{LeLw} = 3000$ km. The coloured					
in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for (1, 2, 3, 4, 5, 6) $\approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the trilinear constellation and before the equilateral triangle constellation, as e.g constellation 6	1099	lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized					
1101 $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation1102(constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed1103according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the1104trilinear constellation and before the equilateral triangle constellation, as e.g constellation 6	1100	in the simulations are added for the following times: (a) $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$ days; (b) for					
1102(constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed1103according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the1104trilinear constellation and before the equilateral triangle constellation, as e.g constellation 6	1101	$(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$ days. When they appear after the equilateral triangle constellation					
1103according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the1104triliniear constellation and before the equilateral triangle constellation, as e.g constellation 6	1102	(constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed					
triliniear constellation and before the equilateral triangle constellation, as e.g constellation 6	1103	according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the					
	1104	triliniear constellation and before the equilateral triangle constellation, as e.g constellation 6	21				

1105Figure 11Intervortical distances of the N=3 point vortex system of an exemplary simulation with friction as in Zhu and1106Cheng (2010). Initial set-up of the distances was  $(l_{LeLw}, l_{HLe}, l_{HLw}) = (2981, 1995, 2000)$  km. Random numbers1107were drawn from a normal Gaussian distribution of zero mean and standard deviation sd = 30 km using R function1108set.seed(12345) in order to estimate the Brownian motion. The other initial conditions are described in the text.21

# 1109 List of Tables

1110	Table 1	Results of the multiple linear regression for Omega blocking as characterized with the trapezoid method. The	
1111	$\alpha$ va	lues show the coefficients of the linearised point vortex equations and the $\beta$ values denote the estimates from	
1112	the l	inear regression. Small p-values indicate more significant regression estimates	22



*Fig. 1.* Schematic illustration of the interaction of three point vortices arranged according to the atmospheric Omega pattern, where the circles indicate the direction and relative strength of rotation. The dotted arrows represent the influence of the other two vortices on the velocity of the corresponding point vortex. Their vector addition given by the solid lines represents the resulting velocity vector for the corresponding vortex. The anti-cyclonic vortex (red) is assumed to be twice as strong as the cyclonic vortices (blue), therefore the induced velocity field is stronger. This interaction can also be derived from Equations 2.



*Fig.* 2. (Left) Two exemplary blocking events, one resembling an Omega (top) and the other a high-over-low (bottom). Shown are the vorticity (coloured) and the geopotential height isolines (grey isolines in 8 dm intervals, bold line represents the 552 dm line) at 500 hPa. (Right) Illustration how the corresponding blocking can be realized in the point vortex model. Upper right figure by courtesy of Müller et al. (2015).



(a) Applying the contour method on an exemplary blocking period



*Fig. 3.* Illustration of the contour method. (a)  $\zeta_{W_k>1}$  (coloured) and geopotential height (grey isolines in 8 dm intervals, bold line represents the 552 dm line) fields for an exemplary blocking period (mean field). The blue and red contours enclose regions with significantly positive and negative vorticity ( $\alpha = 0.5$ ). The solid black line qualitatively represents the number of timesteps, that the corresponding longitude was blocked during the blocking period. The dots mark the  $IBL_{max}$ . The identified blocking vortices are marked as bold (red and blue) contours and their centres are indicated by the circles with their circulation given in  $10^7 m^2 s^{-1}$ . (b) Flow chart of the contour method.



*Fig. 4.*  $\zeta_{w_k>1}$  and geopotential height fields (as in Figure 3a) for an examplary time step to illustrate the trapezoid method. (a) The contour of the largest high pressure system defines the preliminary trapezoid: The minimum and maximum longitudes of the high contours define the northern boundaries of the trapezoid. The dashed lines mark the latitudes, where the minimum vorticity is used (local minimum) within the blocked longitudes. (b) Adapted trapezoid with resulting circulations (in  $10^7 m^2 s^{-1}$ ) and vortex centres marked as crosses



*Fig.* 5. Composite of (a) all 106 high-over-low blockings and (b) all 141 Omega blockings that were identified from 347 blockings during 1990-2012. The mean positions and circulations (in  $10^7 m^2 s^{-1}$ ) of the identified blocking vortices are marked for the contour method with circles and for the trapezoid method with crosses. The  $\zeta_{w_k>1}$  and geopotential height fields are shown as in Figure 3a.



*Fig. 6.* Scatter plot of the circulations  $[10^8 m^2 s^{-1}]$  (averaged for each blocking period) for the two methods. The dashed line shows the ideal case, the bisecting line. The regression line, the correlation coefficient and the  $R^2$  value are shown for the high. The red points show the circulations of the high, the light/dark blue points those of the western/eastern low.



*Fig.* 7. Histogram of the distances *l* between the vortices (a-c) and the circulations  $\Gamma$  (d-f) of the single time steps for Omega blocking as analysed with the (b,e) contour method and (c,f) trapezoid method, and (a,d) high-over-lows as analysed with the contour method. Due to overlapping distributions, the colors accordingly appear darker.

HIRT ET AL.



*Fig. 8.* Scatter plot of the velocities  $u_{\Delta}$  and  $u_d$  with the zonal mean zonal velocity  $\bar{u}$  averaged over  $20 - 80^{\circ}N$ . The grey dashed line indicates the bisecting line, the blue line shows the linear regression. Error intervals (only for Omegas) show the velocities when using the minimum and maximum distances between the vortices instead of the average distance.



*Fig. 9.* Phase space of relative distances  $l_{ij}$ . The fixed point is marked as red cross and the three eigenvectors are displayed as green (stable), blue (unstable) and grey (neutral) lines. Three exemplary trajectories are displayed as points. The elapsed time between two consecutive points corresponds to 8h. The initial condition is marked as star in corresponding colour. Note that the grey trajectory lies on the neutral eigenvector at the initial position and is therefore stationary.





*Fig. 10.* Simulations of two N=3 point vortex systems applying realistic atmospheric conditions. The initial triangles (1) are disturbed from the relative equilibrium of the equilateral triangle of side length l = 2000 km. The distance between the two lows is (a) decreased with  $l_{LeLw} = 1800$  km, (b) increased with  $l_{LeLw} = 3000$  km. The coloured lines mark the trajectories of the corresponding point vortices. Some exemplary triangle constellations 1-6 as realized in the simulations are added for the following times: (a)  $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.5, 2.9, 5.9, 8.8, 14.0)$  days; (b) for  $(1, 2, 3, 4, 5, 6) \approx (0.0, 1.0, 6.2, 12.4, 18.5, 25.0)$  days. When they appear after the equilateral triangle constellation (constellation 1 and 4) and before the trilinear constellation (constellations 3 and 5), the triangles are changed according to the unstable direction, as e.g. constellation 2. Triangles, changed in the stable direction exist after the trilinear constellation and before the equilateral triangle constellation 6.



*Fig. 11.* Intervortical distances of the N=3 point vortex system of an exemplary simulation with friction as in Zhu and Cheng (2010). Initial set-up of the distances was  $(l_{LeLw}, l_{HLe}, l_{HLw}) = (2981, 1995, 2000)$  km. Random numbers were drawn from a normal Gaussian distribution of zero mean and standard deviation sd = 30 km using R function set.seed(12345) in order to estimate the Brownian motion. The other initial conditions are described in the text.

predictor	theory $(\alpha)$	regression estimates ( $\beta$ )	p-value
$ \begin{array}{l} \Gamma_{H} \\ \Gamma_{Lw} \\ \Gamma_{Le} \\ l \end{array} $	$\begin{array}{c} -3.7\cdot 10^{-8}m^{-1}\\ 2.2\cdot 10^{-8}m^{-1}\\ 1.8\cdot 10^{-8}m^{-1}\\ -3.2\cdot 10^{-6}s^{-1}\end{array}$	$-3.4 \pm 1.4 \cdot 10^{-8} m^{-1} \\ -0.7 \pm 1.4 \cdot 10^{-8} m^{-1} \\ 2.7 \pm 1.7 \cdot 10^{-8} m^{-1}$	$0.02 \\ 0.64 \\ 0.11$
$l_{HLe} l_{HLw} l_{LeLw}$	$\begin{array}{c} -4.1\cdot 10^{-6}s^{-1}\\ -4.3\cdot 10^{-6}s^{-1}\\ -2.1\cdot 10^{-6}s^{-1}\end{array}$	$\begin{array}{c} 2.0 \pm 1.2 \cdot 10^{-6} s^{-1} \\ 3.4 \pm 1.1 \cdot 10^{-6} s^{-1} \\ -1.2 \pm 0.8 \cdot 10^{-6} s^{-1} \end{array}$	$0.09 \\ 0.75 \\ 0.10$

*Table 1.* Results of the multiple linear regression for Omega blocking as characterized with the trapezoid method. The  $\alpha$  values show the coefficients of the linearised point vortex equations and the  $\beta$  values denote the estimates from the linear regression. Small p-values indicate more significant regression estimates.