A novel approach for unraveling the energy balance of water surfaces with a single depth temperature measurement

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Abstract

The partitioning of solar energy over the Earth’s surface drives weather and climate of the coupled land–ocean–atmosphere system. Over water surfaces, the evolution of water temperatures at a given depth in the mixed layer implicitly contains the signature of surface energy partitioning, and as such it can be used to diagnose the surface energy balance. In this study, we develop a novel numerical scheme by combining the Green’s function approach and linear stability analysis to estimate the water surface energy balance using water temperature measurement at a single depth. The proposed method is capable of predicting water temperature in the mixed layer, and solving for the components of the surface energy budgets with physically based schemes. Evaluation against in situ measurement and the maximum entropy production method demonstrates that this approach is robust and of good accuracy. It is found that performance of the proposed method depends strongly on the accurate estimation of turbulent thermal diffusivity from in situ measurements, which carries information of meteorological and limnological conditions. Without explicitly using wind speed or temperature/moisture gradient, the proposed approach reduces uncertainty and potential error associated with meteorological measurements in estimation of water surface energy balance.

Partitioning of the solar energy into its various components over the Earth’s surface drives the global energy and water cycles. Since water occupies about 71% of the Earth’s total surface area, accurate estimation of the surface fluxes over water (including sensible heat and latent heat fluxes to the atmosphere, and heat transported to subsurface thermal mass) is of fundamental importance not only for limnology and oceanography, but also in numerical simulations of regional and global weather and climatic processes. To predict the energy fluxes over a water surface, a number of methods have been developed during recent decades, which can be broadly categorized into two groups. The first group simply uses land surface models in which turbulent heat fluxes are estimated using bulk transfer formulae (Oleson et al. 2010; Best et al. 2011; Niu et al. 2011). The water surface is treated as non-vegetated land surface in the models, and the accuracy of predicted turbulent heat fluxes is largely determined by the parameterization of the transfer coefficient. Existing parameterization schemes in this group vary in complexity and assumptions, and no single scheme outperforms others under all conditions (Henderson-Sellers et al. 2003). The second group encompasses empirical models derived from regression analysis of in situ measurements (Morton 1983; Granger and Hedstrom 2011). This group of models mainly focuses on prediction of the latent heat flux, and the resulting site-specific relation may not be applicable to water surface under different oceanographic and meteorological conditions. It is noteworthy that the land surface models and empirical models have two limitations in common. First, the estimated heat fluxes are not necessarily constrained by conservation of energy at the water surface. This can lead to a large residual in the surface energy balance, known as the surface energy imbalance closure problem (Leuning et al. 2012). Second, model predictions are strongly affected by multiple meteorological variables, such as wind speed, air temperature, and moisture (Kiehl and Trenberth 1997). Measurement error related to individual variables is amplified and necessarily leads to great uncertainties in the modeled fluxes.

Developed from different perspectives, two recently proposed numerical methods for estimating surface energy budgets have been able to overcome the aforementioned limitations. The

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methods are capable of predicting surface energy budgets without explicitly using temperature and moisture gradients, wind speed, and empirical parameters, while inherently satisfying the conservation of energy at the interface. The first method is based on the maximum entropy production (MEP) principle (Wang et al. 2014), which predicts the surface energy budget as the most probable and macroscopically reproducible distribution that produces the maximum entropy with given information (Dewar 2005; Wang and Bras 2009). Evaluation against in situ measurements verified accuracy of the MEP model over both land and water surfaces (Wang and Bras 2011; Wang et al. 2014). The second method is the numerical procedure developed by Yang and Wang (2014). Using Green’s function approach, the method is able to reconstruct soil thermal field from a single depth soil measurement of either temperature or heat flux (Wang 2012; Wang and Bou-Zeid 2012). Once the heat flux into the subsurface is obtained, the turbulent fluxes into the atmosphere are estimated using linear stability analysis (Yang and Wang 2014). The model’s accuracy and reliability have been assessed using field measurements. Compared with the MEP model, the numerical procedure has better accuracy, especially for the ground heat flux (Yang and Wang 2014); however, it has not yet been tested over water surfaces.

Compared with land surface, surface energy partitioning over a water surface is further complicated due to the penetration and absorption of solar radiation in the water body. Thermal stratification and turbulent mixing in water bodies are also distinct from subsurface heat transfer process in soils. A schematic comparing surface energy balance over water and land surface is shown in Fig. 1. The goal of this study is thus to generalize the numerical procedure, originally developed for land surface via combined Green’s function approach and linear stability analysis, to estimate the water surface energy balance using temperature measurement at a single depth. The model’s performance is tested against in situ measurement over Lake Geneva, Switzerland. The effects of turbulent heat transfer, radiation penetration and absorption are elucidated by a parametric analysis.

**Methodology**

Considering an infinitesimally thin layer at the water surface (Fig. 1), and ignoring the shortwave absorption at the
surface since liquid water is mostly transparent to solar radiation, the surface energy balance can be written as:

$$R^0 = H + LE + Q_0$$  \hspace{1cm} (1)

where $R^0$ is the net longwave radiation, $H$, $LE$, and $Q_0$ denote the sensible heat, latent heat, and heat transported to deeper water at the surface, respectively ($Q_0$ is usually denoted by $Q_0$ over land). Each flux on the right-hand side of Eq. 1 can be considered as a “dissipative” term that consumes energy at the surface and restores the system to “equilibrium.” Shortwave radiation penetrates through the water surface and warms subsurface water layers. Significant warming begins near the surface and heat is propagated into deeper water level driven by turbulent mixing as a result of the surface wind shear. In general, roughly half of the radiation adds the heating source term $s(z, t)$ to Eq. 2 and creates additional complexity for reconstruction of water thermal fields (see Fig. 1). One underlying assumption of Eq. 2 is that $\kappa$ does not change significantly with depths, thus the proposed method requires a relatively constant thermal diffusivity in the study depths of water body. Furthermore, thermal diffusivity is enhanced through turbulent mixing in the water (Wüst and Lorke 2003). This will be discussed further in the following section.

Using a Green’s function approach for the canonical heat conduction problem in a finite field, the general solution for temperature resulting from Eqs. 2 to 4 is (Cole et al. 2010):

$$T(z, t) = \int_{z=0}^{D_m} g(z, t|z', 0)T(z')dz' + \int_t^{+\infty} \Delta_t \left[ \int_{z=0}^{D_m} g(z, t|z', \tau)s(z', \tau)dz' \right] d\tau$$  \hspace{1cm} (5)

where $z'$ and $\tau$ are integration variables, $g(z, t|z', \tau)$ is the impulse Green’s function solution corresponding to an influx of heat with unity strength (mathematically represented as a Dirac delta function at the surface):

$$g(z, t|z', \tau) = \frac{1}{\sqrt{4\pi\kappa(t-\tau)}} \left[ e^{-\frac{(z-z')^2}{4\kappa(t-\tau)}} + e^{-\frac{(z+z')^2}{4\kappa(t-\tau)}} \right]$$  \hspace{1cm} (6)

On the right-hand side of Eq. 5, the first, second, and third terms represent the contribution of the initial conditions, source term, and boundary conditions to the temperature variability, respectively. Combination of the first and third terms represents a homogeneous heat conduction problem over land surface, whose solution is given by Wang and Bou-Zeid (2012):

$$\int_{z=0}^{D_m} g(z, t|z', 0)T_i(z')dz' + \int_t^{+\infty} \Delta_t \left[ \int_{z=0}^{D_m} \frac{\kappa}{\lambda}T_i(z')dz' \right] d\tau = T_i(z)$$  \hspace{1cm} (7)

where $h(z, \tau)$ is the step Green’s function solution, which resolves the singularity of the impulse Green’s function by temporal integrations (Wang et al. 2005):

$$h(z, \tau) = \frac{\kappa}{\lambda} \int_{\tau=0}^{t} g(z, \tau)d\tau = \frac{2\sqrt{\kappa \tau / \pi}}{\lambda} \exp \left( -\frac{z^2}{4\kappa \tau} \right) - \frac{z}{\lambda} \text{erfc} \left( \frac{z}{2\sqrt{\kappa \tau}} \right)$$  \hspace{1cm} (8)

Therefore, the major outstanding challenge is to solve the contribution of source term $s(z, t)$ to the solution of water temperature. According to the Beer–Lambert law (Jerlov 1976), the intensity of solar radiation decreases.
where:

\[
A_j(z, \tau) = \frac{e^{\tau \mu_j} - e^{-\mu_j \tau}}{2k \mu_j} \text{erfc} \left( \frac{2\kappa \tau \mu_j - z}{\sqrt{4\kappa \tau}} \right) + \frac{e^{\tau \mu_j} + e^{-\mu_j \tau}}{2k \mu_j} \text{erfc} \left( \frac{2\kappa \tau \mu_j + z}{\sqrt{4\kappa \tau}} \right) + \frac{z}{k \mu_j} \text{erf} \left( \frac{z}{\sqrt{4\kappa \tau}} \right) + \sqrt{\frac{4\tau}{k \mu_j}} e^{-z^2/4\kappa \tau}.
\]

In Eq. 7, the boundary condition at the surface yields \( f(t) \equiv Q_0(t) + R^n_s(t) \). Therefore the solution of water temperature at any depth is given by:

\[
T(z, t) = T_s(z) + \int_0^t \left[ Q_0(t-\tau) + R^n_s(t-\tau) \right] d\tau + \int_0^t \frac{R^n_s(t-\tau)}{C_w} \sum_{k=1}^{n-1} \frac{\eta_j \mu_j}{(n-k+1) + R^n_s(n-k) |\Delta A_j(z, k)|} \frac{1}{\Delta h_z(k)}
\]

A numerical quadrature is needed here to discretize the integrand for explicitly formulating the surface temperature \( T_s \) (which is taken here as the temperature of the top few millimeters in water) in terms of water temperature at an arbitrary depth. Discretizing the time continuum into \( t = t_k \) \( k = 0, 1, 2, \ldots, n \) where \( n \) is the number of time steps, and applying the trapezoidal rule to Eq. 12, we obtain:

\[
Q_0(n) = \left\{ 2T(z_n) - J_{n-1}(Q_0, \Delta h_z) - J_n(R^n_s, \Delta h_z) \right\}
\]

\[
- \sum_{k=1}^{n-1} \frac{\eta_j \mu_j}{C_w} \frac{[R^n_s(n-k+1) + R^n_s(n-k)] |\Delta A_j(z, k)|}{\Delta h_z(k)} \frac{1}{\Delta h_z(k)}
\]

where:

\[
J_{n-1}(Q_0, \Delta h_z) = Q_0(n-1) |\Delta h_z(1)|
\]

\[
+ \sum_{k=2}^{n-1} [Q_0(n-k+1) + Q_0(n-k)] |\Delta h_z(k)|
\]

\[
J_n(R^n_s, \Delta h_z) = \sum_{k=2}^{n} [R^n_s(n-k+1) + R^n_s(n-k)] |\Delta h_z(k)|
\]

where \( T(z, n) = T(z) - T_0(z) \) is a normalized temperature, \( \Delta h_z(k) = h_z(t_k) - h_z(t_{k-1}) \) and \( \Delta A_j(z, k) = A_j(z, t_k) - A_j(z, t_{k-1}) \). It is clear from Eq. 13 that \( Q_0(t) \) can be obtained from known quantities, including time series of \( Q_0(t' < t) \) (prior to the current time step), measured time series of water temperature \( T(z, t) \), and net shortwave radiation at the water surface \( R^n_s(t) \). Once the time series of \( Q_0(t) \) is estimated, water temperature at any depth can be obtained by substituting \( Q_0(t) \) into Eq. 12 with prescribed water depth. Similarly, the Green’s function approach enables reconstructing time series of \( Q \) at any depth from a single depth measurement of \( Q(z, t) \) in the mixed layer (Wang 2012).

After time series of \( Q_0 \) and \( T_s \) are obtained, the sensible and latent heat fluxes can be estimated via linear stability analysis. Linear stability analysis estimates turbulent heat fluxes by quantifying their relative efficiencies in restoring the thermodynamic equilibrium of surface energy balance when a perturbation is imposed (Bateni and Entekhabi 2012). Detailed information of the method can be found in previous studies (Bateni and Entekhabi 2012; Yang and Wang 2014). Using linear stability analysis, the relative efficiency of LE to \( H \) for a saturated surface in the absence of advective effects in the atmosphere, i.e. when air near the

<table>
<thead>
<tr>
<th>Wavelength portion of the spectrum (10⁻⁶ m)</th>
<th>Absorption coefficient, ( \mu ) (m⁻¹)</th>
<th>Fraction of radiation, ( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2–0.6</td>
<td>2.874 × 10⁻²</td>
<td>0.2370</td>
</tr>
<tr>
<td>0.6–0.9</td>
<td>4.405 × 10⁻¹</td>
<td>0.3600</td>
</tr>
<tr>
<td>0.9–1.2</td>
<td>3.175 × 10⁻¹</td>
<td>0.1790</td>
</tr>
<tr>
<td>1.2–1.5</td>
<td>1.825 × 10⁻²</td>
<td>0.0870</td>
</tr>
<tr>
<td>1.5–1.8</td>
<td>1.201 × 10⁻³</td>
<td>0.0800</td>
</tr>
<tr>
<td>1.8–2.1</td>
<td>7.937 × 10⁻³</td>
<td>0.0246</td>
</tr>
<tr>
<td>2.1–2.4</td>
<td>3.195 × 10⁻²</td>
<td>0.0250</td>
</tr>
<tr>
<td>2.4–2.7</td>
<td>1.279 × 10⁻³</td>
<td>0.0070</td>
</tr>
<tr>
<td>2.7–3.0</td>
<td>6.944 × 10⁻⁴</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

**Table 1.** Summary of absorption coefficients and radiation fractions for different portions of the solar spectrum in water (Paulson and Simpson 1981).
water surface is in equilibrium with that surface, is given by Yang et al. (2013):

$$\Delta_{re} = \frac{LE}{H} = \frac{L_v}{c_p} \left( \frac{\partial q_s}{\partial T} \right)_{T=T_s}$$

(16)

where $c_p$ is the specific heat of air; $L_v$ is the latent heat of vaporization for water; $q_s$ is the saturated specific humidity. Using the Clausius–Clapeyron equation, $q_s$ is a function of surface temperature and therefore $\Delta_{re}$ depends only on $T_s$. Detailed derivation of $\Delta_{re}$ is provided in Yang and Wang (2014). The sensible and latent heat flux are then given by:

$$H = \frac{R_n}{\Delta_{re} + 1} \quad \text{and} \quad LE = \frac{\Delta_{re}}{\Delta_{re} + 1} \left( R_n - Q_0(n) \right)$$

(17)

It is clear from Eqs. 13 and 17 that the proposed method only requires the surface radiation budget and water temperature at a single depth as input. The water surface fluxes are formulated without explicitly using meteorological variables such as wind speed, air temperature, and humidity. This reduces the error and uncertainty of the modeled surface fluxes since the measurement error of radiation budget and temperature are significantly smaller than that of wind speed and temperature/moisture gradient (Wang et al. 2014). Furthermore, the model prediction of the fluxes is bounded by the conservation of energy at the surface. The model however requires equilibrium between the air and water layers near the interface (weak horizontal heat transfer in both layers); turbulent fluxes are computed at a rate dissipating radiative energy to maintain that surface equilibrium.

In a nutshell, the numerical method for estimating the water surface energy balance developed in this study follows a two-step procedure. First, heat transported to deeper water at the surface $Q_0$ is estimated by solving one-dimensional heat transfer problem via Green’s function approach. Using Eqs. 2-4 for heat transfer in the mixed layer, the method assumes that lateral boundary conditions and horizontal heat transport play a negligible role in determining water temperatures. Hence, the developed method is preferably applicable for large water bodies with homogeneous or weak horizontal heat transport, and far from water–land boundaries. Estimating $Q_0$ in moving water bodies with complex lateral boundaries (e.g., rivers) will require adding a horizontal heat source term on the right-hand side of Eq. 2 as well as a reasonable description of the lateral boundary conditions. Moreover, turbulent fluxes $H$ and $LE$ are predicted based on the $Q_0$ estimated in the first step and the measured net longwave radiation $R_n^*$.

The major underlying assumptions of the proposed method include: (1) the quasi-static thermal equilibrium between the water surface and near-surface atmosphere, inherited from the premise of the linear stability analysis (Bateni and Entekhabi 2012), and (2) the analogy in turbulent heat and moisture transport (Yang and Wang 2014). Approaching the thermal equilibrium, the evolution of the state of near-surface air (i.e., temperature, humidity, etc.) follows closely that of the surface (Bateni and Entekhabi 2012) with its signal embedded in the evolution of water surface state. In addition, the analogy in transport mechanism of heat and moisture is commonly assumed in the literature when in situ measurements are unavailable (Liang et al. 1994; Mote and O’Neill 2000), canceling the dependence of the $LE/H$ ratio on the aerodynamic resistance. Subsequently, $H$ and $LE$ can be formulated without explicitly using meteorological variables based on these assumptions. Using flux measurements over the ocean, Large and Pond (1982) found that transfer coefficients of moisture and heat are very similar under various meteorological conditions as long as the atmospheric layer is unstable. However, the assumptions do not necessarily hold all the time (Stensrud 2009). For instance, the transfer coefficient of heat is found to be significantly smaller than that of moisture in a stable air layer (Large and Pond 1982). Wallace and Hobbs (2006) showed that the magnitude of transfer coefficients of moisture and heat diverges as wind speed increases. Strong horizontal air movement can also break the equilibrium between water surface and near-surface air. Therefore, better accuracy of the current method is expected in predicting surface turbulent heat fluxes when applied to unstable atmospheric conditions with low wind speeds.

**Site description**

In this study, the proposed method is tested against in situ measurements over Lake Geneva, Switzerland. The field data were collected during the Lake-Atmosphere Turbulent Exchange (LATEX) field campaign from 15 August to 27 October 2006. The experimental platform was located about 100 m offshore in a shallow part of the lake without significant aquatic vegetation (Vercauteren et al. 2008). Meteorological variables such as wind velocity, air temperature, and humidity were measured at four different heights above the water surface using a vertical array of sensors. In this study, we used the measurements at 1.66 m for subsequent analysis, which is the closest to the water surface. A water temperature profile was measured at a frequency of 5 min using a Raman-scattering fiber-optic temperature profiler. About 1 m of the profiler was above the water surface, whereas the remaining 1.85 m was submerged in water. The vertical and temperature resolution of the profiler is 0.004 m and 0.01°C. Due to technical issues, several gaps exist during the measurement period. Detailed information on equipment and experimental setup can be found in Vercauteren et al. (2008, 2011).

**Results and discussion**

**Sensitivity analysis**

Penetration and absorption of shortwave radiation are the major players that distinguish surface energy partitioning
and subsurface heat transfer for water bodies from their land counterparts. From Eq. 9, it is clear that radiative absorption is mainly affected by the available net shortwave radiation at the water surface $R_n^s(t)$ and the absorption coefficient $\mu$. For the study site, Vercauteren et al. (2011) found that the effective thermal diffusivity in top 1.5 m of the lake mainly depended on turbulence, and hence was much larger here than molecular diffusivity. By analyzing phase shift and amplitude of variation of water temperature, they estimated an effective thermal diffusivity ranging from $6.6 \times 10^{-5}$ to $3.0 \times 10^{-3}$ m$^2$ s$^{-1}$ in sunny conditions with low wind speeds. Thermal diffusivity determines the heat transfer rate and has a significant effect on the temperature solution. Thus, we conducted a parametric analysis to quantify the sensitivity of water temperature to $R_n^s(t)$, $\mu$ and $\kappa$. The simulation starts at 00:00 and ends at 24:00 on 4 September 2006, a sunny day with low wind speeds. As shown in Eqs. 12 and 13, constructing the subsurface thermal field requires $R_n^s(t)$ and temperature measurement at a single depth as input. Here, the measured water surface temperature from the fiber-optic temperature profiler is used. To simplify the problem in the parametric analysis, a uniform initial temperature profile is assumed, and net shortwave radiation at the water surface is prescribed by a sinusoidal function: 

$$R_n^s(t) = A \sin(\omega t_{\text{loc}} + \phi), \quad 6 \leq t_{\text{loc}} \leq 18$$

(18)

where $A$ is the amplitude of diurnal variation, $\omega = 2\pi/24$ rad h$^{-1}$ is the angular speed of rotation of the earth, $t_{\text{loc}}$ is the local time in hours, $\phi$ is the phase lag. Three sets of simulations were carried out separately to illustrate the impact of $R_n^s(t)$, $\mu$, and $\kappa$ on the vertical profile of water temperature. Four different amplitudes were tested in the first set, namely, 0, 200, 400, and 600 W m$^{-2}$, while $\mu$ is adopted from Table 1 and a mean thermal diffusivity of $1.0 \times 10^{-4}$ m$^2$ s$^{-1}$ is used for $\kappa$. For the second set, $\mu$ is multiplied by a parameter $x$ to represent the absorption process with different water characteristics. Tested $x$ ranges from 0.5 to 4, and the amplitude $A$ and thermal diffusivity $\kappa$ have a constant value of 400 W m$^{-2}$ and $1.0 \times 10^{-4}$ m$^2$ s$^{-1}$, respectively, in these runs. In the last set, $\kappa$ is varied between $1 \times 10^{-6}$ and $1 \times 10^{-3}$ m$^2$ s$^{-1}$, while $\mu$ is adopted from Table 1 and $A$ is fixed to 400 W m$^{-2}$.

Vertical profiles of predicted water temperature with varying $A$ at 12:00 and 16:00 are shown in Fig. 2a,b. Without shortwave radiation absorption, heat transfer over the water surface is identical to that over the land surface. The profile with $A = 0$ represents the temperature solution driven by homogeneous heat conduction, where the surface forcing is embedded in the time series of surface temperature. As net longwave radiation at the surface is the sole energy source, water temperature decreases with depth. When there is shortwave radiation absorption, it provides additional energy other than surface forcing that heats up water unevenly in the vertical direction, which modifies the homogeneous heat transfer process in the water body. The difference between various temperature profiles in Fig. 2a,b thus represents the cumulative effect of heat transfer and radiation absorption at different times. Fig. 2a,b clearly show that the water temperature increases with the intensity of shortwave radiation, as expected. At 12:00, the maximum temperature increase is found at the depth of 0.5 m. Compared with the case with no shortwave radiation, temperature increase is about 0.28, 0.55, and 0.83°C for $A = 200$, 400, and 600 W m$^{-2}$, respectively. The vertical distribution of temperature increase has a linear relationship with the magnitude of $R_n^s(t)$, as indicated in Eq. 12. Note that with a stronger shortwave radiation, water temperature at a few centimeters depth can be slightly greater than the surface temperature, owing to the large absorption coefficient at the longer end of the solar wavelength band, as presented in Table 1. At 16:00, the maximum temperature (compared with the no radiation case) increase occurred at the depth of 0.9 m. The shift of depth where the maximum temperature increase occurs is due to reduced $R_n^s$ after noontime. The effect of radiation absorption decreases with reduced $R_n^s$ as heat transfer plays a more important role in determining water temperature. As the temperature closer to the surface is consistently higher with decreased radiation intensity, diffusion tends to transfer the large amount of accumulated surface thermal energy to deeper water. At the depth of 1.5 m, shortwave radiation of 600 W m$^{-2}$ will lead to a temperature increase of about 0.51°C at 12:00 and about 0.72°C at 16:00.

Results with different $x$ values at 12:00 and 16:00 are plotted in Fig. 2c,d. By changing $x$, the vertical absorption of shortwave radiation in the water body is modified, leading to a different vertical distribution of temperature. However, it is clear from Fig. 2c,d that the temperature profiles are insensitive to the selected $x$ values. At 12:00 and 16:00, the maximum temperature difference is less than 0.1°C when $x$ is increased from 0.5 to 4. The negligible impact is caused by the efficiency of turbulence in transferring energy in the mixed layer, which redistributes (mixes) the local effect of modified radiation absorption rapidly. This is a useful result since it indicates that our model skill as assessed in the following section is minimally sensitive to $\mu$, which is a parameter that can vary across sites and with water conditions as explained before (except potentially in sites with very high turbidity).

The impact of $\kappa$ on water temperature solutions at 12:00 and 16:00 is demonstrated in Fig. 2e,f. It is shown that the vertical distribution of water temperature changes markedly with the magnitude of thermal diffusivity $\kappa$. With a small $\kappa$ value of $1 \times 10^{-6}$ m$^2$ s$^{-1}$, heat transfer in the water is so slow that absorption of shortwave radiation essentially dictates the profile. At 12:00, the maximum temperature is found at the depth of 0.05 m, and temperatures deeper than 0.6 m remain mostly unperturbed (warming <0.2°C as
compared with initial temperature). Results constructed from $T_{0,10}$ were very close to results shown in the manuscript. A larger $j$ value represents a faster heat transfer in the water body, and therefore tends to smooth the vertical temperature profile. When $j$ is $1 \times 10^{-4}$ m$^2$ s$^{-1}$, large thermal diffusivity transports heat very efficiently, such that absorbed

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Vertical profile of predicted water temperature with (a) varying $A$ at 12:00, (b) varying $A$ at 16:00, (c) varying $\alpha$ at 12:00, (d) varying $\alpha$ at 16:00, (e) varying $\kappa$ at 12:00, and (f) varying $\kappa$ at 16:00, on 4 September. Reference values of parameters, unless varied and specified in the graph, are $A = 400$ W m$^{-2}$, $\alpha = 1$, $\kappa = 1.0 \times 10^{-4}$ m$^2$ s$^{-1}$.
\end{figure}
energy is redistributed evenly throughout the simulated depths. At 12:00, the difference between surface temperature and water temperature at a depth of 1.5 m is only about 0.6°C. Temperature profiles with different \( \kappa \) values at 16:00 are qualitatively similar to those at 12:00. After 4 hours of heat transfer, under the condition of small thermal diffusivity (\( \kappa = 1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1} \)), the depth exceeds which water temperature remains mostly uninterrupted (warming < 0.2°C as compared with initial temperature) shifts from 0.6 m to 0.9 m. The parametric analysis indicates that intensity of the net shortwave radiation \( R_{\text{ns}}(t) \) and thermal diffusivity \( \kappa \) determine the distribution of temperature increase, while the absorption coefficient \( \mu \) has a negligible impact under a turbulent heat transfer regime.

**Reconstruction of water temperature**

Following the previous study (Yang and Wang 2014), temperature measurement at 0.05 m depth from fiber-optic temperature profiler is adopted to construct the water thermal field. The simulation period is from 1 September to 17 October 2006. The depth of 0.05 m is selected for the following reasons: (1) it is in the thermally active shallow water layer such that the surface energy balance signal is not significantly contaminated by numerical errors and instabilities in thermal reconstruction; and (2) it is not too close to the surface such that the signal is not largely influenced by the oscillation of water surface conditions (e.g., waves). Nevertheless, we performed the reconstruction using a depth of 0.1 m and the results were very similar to the ones we report; the reconstruction is thus not very sensitive to the exact depth of the measurements. Measured net radiation and wind speed during the study period are shown in Fig. 3.

It is clear from the graph that the lake is under sunny condition with low wind speeds in general. Vercauteren et al. (2008) reported that the heights of the waves in the lake had a median of about 0.03 m and rarely exceeded 0.2 m for wind speeds between 1 and 10 m s\(^{-1}\).

Vertical mixing in the mixed layer is mainly caused by wind stress on the water surface such that \( \kappa \) is not expected to be constant in time and space. By analyzing phase shift and amplitude of variation of water temperature in the lake, Vercauteren et al. (2011) found that high wind speed tends to increase depth of the mixed layer, as it generates stronger shear force at the surface. During September, it was observed that the top 1 m of the lake is generally well mixed with an estimated \( \kappa \) of about \( 1.2 \times 10^{-4} \text{ m}^2 \text{ s}^{-1} \). Hence, we used a
constant $\kappa$ of $1.2 \times 10^{-4}$ $\text{m}^2\text{s}^{-1}$ for reconstructing water temperatures in top 1 m of the lake. It is noteworthy that our approach is capable of reconstructing water temperatures with varied $\kappa$, as long as temporal variation of $\kappa$ is available. The diffusivity estimated from field measurements implicitly contains information of mean atmospheric conditions and water column stability in this period. Thus the proposed method here is not entirely uncoupled from atmospheric conditions.

Reconstructing water temperature requires measured net shortwave radiation as input, while only net radiation is available for this particular dataset. To estimate the net shortwave radiation, the Stefan-Boltzmann law is used:

$$R_n^\text{a} = R_n - R_n^w = (\varepsilon_a T_a^4 - \varepsilon_w T_w^4)$$  \hspace{1cm} (19)

where $\varepsilon_a$ and $\varepsilon_w$ denote the emissivity of air and water; $\sigma$ is the Stefan-Boltzmann constant $5.67 \times 10^{-8}$ $\text{W m}^{-2}\text{K}^{-4}$, and $T_a$ is the air temperature in Kelvin. The emissivity of water $\varepsilon_w$ is relatively constant; here we used a value of 0.98 following a previous study (Rees and James 1992). The atmospheric emissivity $\varepsilon_a$ in Eq. 19 represents the whole atmospheric column of air above the site; it is thus sensitive to various meteorological variables, including air temperature, relative humidity, cloudiness, etc. (Crawford and Duchon 1999), and ranges from the clear-sky value to the fully cloudy value of unity. Following a previous study (Satterlund 1979), $\varepsilon_a$ is calculated using:

$$\varepsilon_a = \text{clf} + (1 - \text{clf}) \times \left\{ 1.08 \left[ 1 - \exp \left( -e_0T_a^{2016} \right) \right] \right\}$$  \hspace{1cm} (20)

where clf is the cloud fraction, $e_0$ is the vapor pressure in millibar. A clear-sky condition is assumed in this study as cloud fraction cannot be estimated without direct measurement of downward shortwave radiation. Estimated $\varepsilon_a$ from the Eq. 20 falls in a small range of 0.84-0.88 for the simulation period over the study site. Considering the air temperature range of 10-20°C during the study period, maximum error in estimated $R_n^\text{a}$ and $R_n^w$ is about 51.0-58.5 $\text{W m}^{-2}$. In addition, in the previous section, the sensitivity analysis showed that a difference of 200 $\text{W m}^{-2}$ in magnitude of $R_n^\text{a}$ leads to a difference of about 0.28°C for water temperatures in the top 1.5 m. Therefore, the error due to $\varepsilon_a$ estimation is tolerable for the purpose of water temperature reconstruction in this study.

The air temperature measured at 1.66 m above the water surface is used for Eq. 19 as it is the closest to the surface. Three sets of surface temperature data are available, collected by thermocouple, IR sensor, and fiber-optic temperature profiler, respectively. Measurement from fiber-optic temperature profiler is selected, as it has the highest data availability and is consistent with the input temperature measurement. As water level varies with wind condition throughout the day, surface temperature is determined by finding the sharp change around still water level in the temperature profile, where difference between two adjacent temperature measurements exceeds 2.5 times of the standard deviation of water temperature.

Comparisons of model predicted surface temperature ($T_s$), water temperature at 0.15 m ($T_{0.15}$), and water temperature at 1.0 m ($T_{1.0}$) against field observations are shown in Fig. 4. Due to gaps in measured input variables, predicted $T_s$, $T_{0.15}$ and $T_{1.0}$ are discontinuous. The agreement between prediction and observation is generally good throughout the simulation period, confirming the model’s capability in accurately reconstructing water temperature from measurements at another depth in the mixed layer. For $T_{0.15}$ and $T_{1.0}$, the coefficient of determination $R^2$ is 0.98 and 0.96, and root mean square error (RMSE) is less than 0.1°C, indicating a reasonable goodness-of-fit between predictions and observations. It is found that the model’s accuracy of water surface temperature prediction is slightly impaired, as compared with the prediction of water temperature at 0.15 m and 1.0 m. This could be related to the variability of boundary conditions at the water–air interface, e.g., wave, wind shear, boat wakes, etc. Furthermore, Vercauteren et al. (2009) reported that a considerable deviation existed between surface temperature measurements from different sensors. Evaluating the predictions of surface temperature thus becomes challenging with the significant uncertainty and error related to in situ measurements.

In addition, it is also important to evaluate the performance of the method in predicting spatial distribution of water temperature. Simulated water temperature profiles are compared against field measurements in the lake for two days with distinct meteorological conditions: a calm clear day (maximum wind speed < 1 m s$^{-1}$), September 20 (Fig. 5a), and a windy day (maximum wind speed > 5 m s$^{-1}$), October 8 (Fig. 5b). Though it is aforementioned that the proposed method is preferably applicable for low wind conditions, Fig. 5 shows that reconstructed temperature profiles agree with measurements reasonably well for the sunny day and the windy day. Note that the initial time of simulation for the sunny day and the windy day is 10:00, September 17 and 06:00, October 6, respectively. A full diurnal cycle (warming and cooling of water temperatures) has been simulated before the predicted temperature profiles shown in Fig. 5 to reduce the impact of initial conditions. We should emphasize here that the information of mean atmospheric and water conditions, rather than atmospheric and water variability, is implicitly contained in the estimated diffusivity from field measurements. Thus, it does not contradict the reasonable agreement in Fig. 5 with a same $\kappa$ for two distinct meteorological conditions. At depths between 0.1 and 1.0 m, difference between predicted and observed water temperatures is less than 0.2°C. The largest deviation of more than 2°C is observed at the surface, which is mainly attributable to measurement errors and fluctuation of boundary.
conditions at the water–air interface. In the vertical direction, surface temperature is the highest (compared with other depths) during daytime (12:00 and 16:00) and the lowest at night (4:00 and 20:00) on the calm clear day. On the other hand, the surface is consistently the coolest point of the profile throughout the diurnal cycle on the windy day. This indicates that presence of solar radiation has a significant warming effect on water layer close to the surface (<0.1 m) only under a low wind condition.

**Prediction of surface energy balance**

After the time series of surface temperature is reconstructed, H and LE can be estimated from Eq. 17. Throughout the simulation period, both observed and predicted
turbulent heat fluxes contain several big gaps due to limited data availability. Hence scatter plots are used to compare model prediction against observation over the entire simulation period. Note that due to complexities at the water–air interface, direct in situ measurement of \( Q_0 \) at the water surface is extremely challenging. In practice, the most commonly used (hereafter referred to as “the conventional”) approach to estimate \( Q_0 \) is through a combination of gradient method and calorimetry (Liebethal et al. 2005):

\[
Q_0(t) = -k \frac{\partial T}{\partial z}_{z_q} + \int_{z_0}^{z_q} C_w \frac{\partial T}{\partial z} \, dz - \int_{z_0}^{z_q} C_{ws}(z, t) \, dz \tag{21}
\]

where \( z_q \) is an arbitrary depth close to surface and equals to 0.05 m in this study. This approach accounts for the vertical gradient of temperature measured at \( z_q \) and includes the heat storage in the water body above when determining \( Q_0 \). From Eq. 21 it is clear that the conventional approach uses measured time series of water temperature profiles to calculate \( Q_0 \), while the proposed method in this study (Eq. 13) only requires measured time series of water temperature at a single depth.

Figure 6 shows that predicted \( Q_0 \) from the single-depth method in this study agrees well with that from the conventional approach, while modeled turbulent heat fluxes are not highly accurate. RMSEs for \( H \), \( LE \), and \( Q_0 \) are about 30, 71, and 22 W m\(^{-2} \), respectively. As the model capacity in estimating \( Q_0 \) is validated by Fig. 6c, the substantial discrepancy in \( H \) and \( LE \) is hypothesized as primarily due to the inadequacy of radiation parameterization. The estimate of net longwave radiation entails among other uncertainty in the surface temperature prediction, which is strongly affected by the oscillation of water surface conditions.

**Comparison to the MEP model**

To illustrate the performance of the method proposed in this study with respect to existing numerical approaches, here we compare it with the MEP model. Wang et al. (2014) recently extended the MEP model for surface energy budgets over water, snow, and ice surfaces. Model prediction shows encouraging agreement with observation from several field experiments. The relative efficiency of \( LE \) to \( H \) in the MEP model is given by Wang et al. (2014):

\[
\Delta_{re} = \frac{LE}{H} = 6 \left( 1 + \frac{1111 \times 36}{150} \left( \frac{\partial q'}{\partial T} \right)_{T=T_a} - 1 \right) \tag{22}
\]

and \( Q_0 \) is calculated by a physically based analytical solution:
where \( I_{\text{wei}} = \sqrt{\rho_w c_w \lambda} \) is the thermal inertia parameter of water (Wang et al. 2014), \( \rho_w \) and \( c_w \) denote the density and specific heat of water. Once \( Q_0 \) is obtained, the MEP model can predict turbulent heat fluxes by distributing available energy at the surface (\( R_n^0 - Q_0 \)) based on the relative efficiency between \( H \) and \( LE \) in Eq. 22. Note that the MEP model only requires \( R_n^0 \), \( R_n^0 \), and \( T_s \) as inputs to estimate the surface energy budgets.

Model predictions by the proposed and the MEP methods are shown in Fig. 7. Predicted surface heat fluxes from both methods are in good agreement. A negative \( Q_0 \) value implies transport of thermal energy from water to the atmosphere. This confirms the finding in previous study that shortwave radiation absorption within the water layer close to the surface is an important energy source of the turbulent heat fluxes (Wang et al. 2014). Compared against observed turbulent fluxes and \( Q_0 \) from the conventional approach, the MEP model has RMSEs of about 25, 39, and 36 W m\(^{-2}\) for \( H \), \( LE \), and \( Q_0 \), respectively. These results indicate that the method proposed in this study has a slightly better overall performance than the MEP model, when driven by the accurate measurement of available energy, in predicting water surface...
energy balance over the study site. This finding is consistent with simulation results over land surface in previous study (Wang et al. 2014; Yang and Wang 2014).

**Concluding remarks**

In this study, we developed a new physically based scheme to estimate the surface energy components over water surface by combining the linear stability analysis and the Green’s function approach. The underlying mechanism is that subsurface thermal mass in the mixed layer implicitly contains the signal of surface temperature evolution, which regulates the partitioning of solar energy at the surface. Capable of predicting all dissipative surface energy budgets, the method only requires the net longwave and shortwave radiative fluxes and temperature measurement at a single depth in the mixed layer, and is thus highly suitable for large-scale applications. Performance of the proposed method is strongly affected by estimation of turbulent thermal diffusivity, which carries the information of meteorological and limnological conditions. Without explicitly using wind speed or temperature/moisture gradient, the method substantially reduces the uncertainty and potential error associated with meteorological measurements. Using the estimated \( \kappa \) from in situ measurements, results show that the model is able to reconstruct water temperature profile from a single-depth measurement reasonably well. With an accurate measurement of available energy at the water surface, predicted \( H \) and \( LE \) from the linear stability analysis method are in reasonable agreement with experimental observations. In addition, results of comparison with the MEP model illustrate that the proposed method is of better accuracy over the study site.

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**Fig. 7.** Comparison of predicted (a) \( LE \), (b) \( H \), and (c) \( Q_0 \) from the MEP model and the proposed method for 1 September–17 October 2006.
Though results from the proposed method are promising, there are a few limitations that need to be addressed in future studies. The contribution of aerodynamic conditions in parameterizing turbulent heat fluxes is not accounted in this study. This simplification underestimates the influence of aerodynamic conditions on surface energy partitioning, and may lead to considerable bias under strong wind conditions. A uniform and time-invariant thermal diffusivity in vertical direction is assumed in this study based on previous data analysis (Vercauteren et al. 2011). This particular value is not expected to be universal, and the homogeneity and time-invariance assumptions might not hold in oceans or seas where mixing in the water body can fluctuate significantly more than in a lake. Though our simulation in this study does not strictly account for temporally varied $\kappa$, we recommend time and site-specific determination of turbulent thermal diffusivity, especially for study areas and periods where meteorological and limnological conditions change vastly. Another limitation is that model performance is evaluated with a limited data set of field measurements over a lake. Further tests of the proposed method over water surfaces with different fetch sizes (and over oceans and seas) and hydroclimatic conditions are needed, particularly when the turbulent diffusivity in the water is different from the typical lake value we use here. Nevertheless, the method proposed in this study offers a novel and physically based tool to predict water surface energy partitioning with a minimal set of information. With enhanced representation of water–atmosphere interaction, the method has potential applications for water ecosystems and oceanographic study, especially at large scales.

References


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Conflict of Interest
None declared.